PRINCIPLES of MATHEMATICS 10

Solutions Manual

Series Author and Senior Consultant
Marian Small

Lead Author
Chris Kirkpatrick

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Ian Macpherson • John Rodger • Susanne Trew

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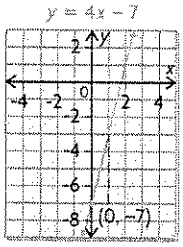
Chapter 1: Systems of Linear Equations

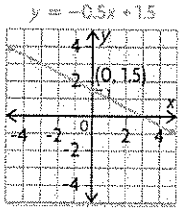
Getting Started, page 4

1. a) The place where graph crosses the x-axis is called the x-intercept, so i) is the correct choice.
b) In the equation, $y = 5x + 2$, 5 is a coefficient of the variable x. The correct choices, in order, are iii), v), and iv).
c) Let $x = 0$ to determine the y-intercept of $y = 4x - 7$. The correct choice is ii).
d) You can determine the solution to $20 = 3x - 10$ by graphing $y = 3x - 10$. The correct choice is vii).
e) The ordered pair at which two lines cross is called the point of intersection. The correct choice is vi).

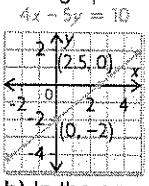
2. a) For $y = 4x - 7$, the y-intercept is -7, and the slope is 4. So the graph goes through points $(0, -7)$ and $(1, -3)$.



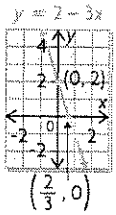
b) For $x + 2y = 3$, the y-intercept is 1.5 and the slope is -0.5. So the graph goes through points $(0, 1.5)$ and $(1, 1)$.



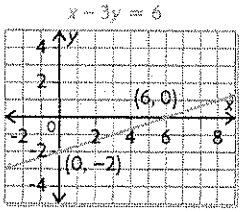
3. a) In the equation $4x - 5y = 10$, when $x = 0$, then $y = -2$, and when $y = 0$, then $x = 2.5$. So, the graph goes through points $(0, -2)$ and $(2.5, 0)$.



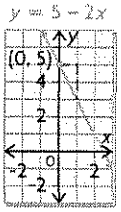
b) In the equation $y = 2 - 3x$, when $x = 0$, then $y = 2$, and when $y = 0$, then $x = \frac{2}{3}$. So, the graph goes through points $(0, 2)$ and $\left(\frac{2}{3}, 0\right)$.



4. a) For example, for $x - 3y = 6$, when $x = 0$, then $y = -2$, and when $y = 0$, then $x = 6$. So, the graph goes through points $(0, -2)$ and $(6, 0)$.



b) For example, for $y = 5 - 2x$, when $x = 0$, then $y = 5$, and when $y = 0$, then $x = 2.5$. So, the graph goes through points $(0, 5)$ and $(2.5, 0)$.



5. a) $5x + 10 - 3x + 12 = 5x - 3x + 10 + 12$
$= 2x + 22$

b) $4(3x + 5) = 4(3x) + 4(5)$
$= 12x + 20$

c) $-2(5x - 2) = -2(5x) - 2(-2)$
$= -10x + 4$

d) $(3x - 6) + (2x + 7) = 3x + 2x - 6 + 7$
$= 5x + 1$

e) $6(2x - 4) - 3(2x - 1) = 6(2x) + 6(-4) - 3(2x) - 3(-1)$
$= 12x - 24 - 6x + 3$
$= 12x - 6x - 24 + 3$
$= 6x - 21$

f) $(8x - 14) + (7x + 6) = 8x + 7x - 14 + 6$
$= 15x - 8$

6. a) From $Ax + By + C = 0$ form to $y = mx + b$ form:

$3x + 4y - 6 = 0$

$4y = -3x + 6$

$y = -\frac{3}{4}x + \frac{3}{2}$

b) From $y = mx + b$ form to $Ax + By + C = 0$ form:

$y = 2x - 5$

$0 = 2x - y - 5$

$2x - y - 5 = 0$

c) From $Ax + By + C = 0$ form to $y = mx + b$ form:

$4x - 7y - 3 = 0$

$-7y = -4x + 3$

$y = \frac{4}{7}x - \frac{3}{7}$

d) From $y = mx + b$ form to $Ax + By + C = 0$ form:

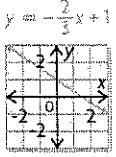
$y = -\frac{2}{3}x - \frac{5}{6}$

$6y = -4x - 5$

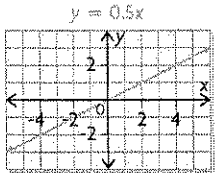
$4x + 6y + 5 = 0$

7. a) The equation $y = 3x - 5$ is in the form $y = mx + b$, with $m = 3$ and $b = -5$. So, the slope is 3 and the y-intercept is -5.

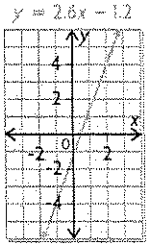
b) The equation $y = -\frac{2}{3}x + 1$ is in the form $y = mx + b$, with $m = -\frac{2}{3}$ and $b = 1$. So, the slope is $-\frac{2}{3}$ and the y-intercept is 1.



c) The equation $y = 0.5x$ is in the form $y = mx + b$, with $m = 0.5$ and $b = 0$. So, the slope is 0.5 and the y-intercept is 0.



d) The equation $y = 2.6x - 1.2$ is in the form $y = mx + b$, with $m = 2.6$ and $b = -1.2$. So, the slope is 2.6 and the y-intercept is -1.2.



8. Relation c) is a direct variation because it is a straight line that passes through the origin.
Relations a), b), and d) are partial variations because they are straight lines that do not pass through the origin.

9. a) The graph shows that at time 0, that is, when Kyle starts out home from school, he is 4.5 km from home. So, the school is 4.5 km from Kyle's home.

b) The graph shows Kyle arrived home 15 min after he started. He travelled 4.5 km in 15 min, which is the same speed as 18 km in 60 min or 1 h. He cycles at 18 km/h.

10. a) The relation $y = 3x - 6$ is linear, because it is of degree 1.

b) The relation is linear, because the first differences are constant, 2.

c) The relation $y = 5x^2 + 6x - 4$ is nonlinear, because it is of degree 2.

d) The relation is nonlinear, because the first differences are not constant.

11. a)
$$\begin{aligned} x + 5 &= 12 \\ x &= 12 - 5 \\ x &= 7 \end{aligned}$$

b)
$$\begin{aligned} 13 &= 9 - x \\ x &= 9 - 13 \\ x &= -4 \end{aligned}$$

c)
$$\begin{aligned} 2x &= 18 \\ x &= 18 \div 2 \\ &= 9 \end{aligned}$$

d)
$$\begin{aligned} -3x &= -21 \\ x &= -21 \div -3 \\ &= 7 \end{aligned}$$

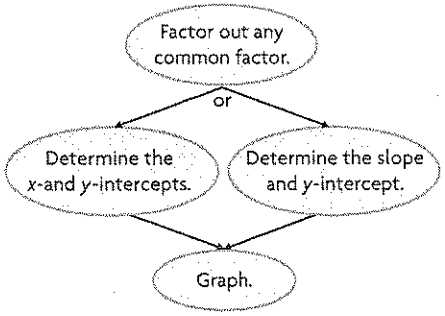
e)
$$\begin{aligned} 2x - 5 &= 15 \\ 2x &= 20 \\ x &= 20 \div 2 \\ &= 10 \end{aligned}$$

f)
$$\begin{aligned} 4x - 6 &= 8x + 2 \\ 4x - 8x &= 2 + 6 \\ -4x &= 8 \\ x &= 8 \div -4 \\ &= -2 \end{aligned}$$

12. a)
$$\begin{aligned} 3x - 2y &= 14 \\ 3(1.5) - 2y &= 14 \\ 4.5 - 2y &= 14 \\ -2y &= 14 - 4.5 \\ y &= (14 - 4.5) \div -2 \\ &\doteq -4.8 \end{aligned}$$

b)
$$\begin{aligned} 0.36x + 0.54y &= 1.1 \\ 0.36x + 0.54(0.7) &= 1.1 \\ 0.36x + 0.378 &= 1.1 \\ 0.36x &= 1.1 - 0.378 \\ x &= 0.722 \div 0.36 \\ &\doteq 2.01 \end{aligned}$$

13. a) For example,



b) For example, I would determine the x- and y-intercepts because I think it's easier.

Lesson 1.1: Representing Linear Relations, page 12

1. Substitute the values of each ordered pair into $2x + 4y = 20$ to see if they make the equations true.

a) For (10, 0):

$$\begin{aligned} LS &= 2x + 4y \\ &= 2(10) + 4(0) \\ &= 20 \\ LS &= RS \end{aligned}$$

(10, 0) is on the graph.

b) For (–3, 7):

$$\begin{aligned} LS &= 2x + 4y \\ &= 2(-3) + 4(7) \\ &= -6 + 28 \\ &= 22 \\ LS &\neq RS \end{aligned}$$

(–3, 7) is not on the graph.

c) For (6, 2):

$$\begin{aligned} LS &= 2x + 4y \\ &= 2(6) + 4(2) \\ &= 20 \\ LS &= RS \end{aligned}$$

(6, 2) is on the graph.

d) For (0, 5):

$$\begin{aligned} LS &= 2x + 4y \\ &= 2(0) + 4(5) \\ &= 20 \\ LS &= RS \end{aligned}$$

(0, 5) is on the graph.

e) For (12, –1):

$$\begin{aligned} LS &= 2x + 4y \\ &= 2(12) + 4(-1) \\ &= 20 \\ LS &= RS \end{aligned}$$

(12, –1) is on the graph.

2. a) For example, I started with no muffins, which meant Jacob could spend \$15 on doughnuts. At 25¢/doughnut, than means Jacob can buy 60 doughnuts. Then I increased the number of muffins he would buy. I increased the number by 4 because that made for an easy multiple. I calculated the number of doughnuts he could get each time.

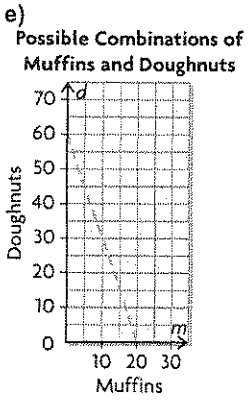
Muffins		Doughnuts		Total
Number	Cost (\$)	Number	Cost (\$)	Cost (\$)
0	0.00	60	15.00	15.00
4	3.00	48	12.00	15.00
8	6.00	36	9.00	15.00
12	9.00	24	6.00	15.00
16	12.00	12	3.00	15.00
20	15.00	0	0.00	15.00

b) To get the maximum number of muffins, Jacob would have to spend all the money on muffins and none on doughnuts. One muffin costs 75¢, so the number he can buy for \$15 is $15 \div 0.75 = 20$.

c) To get the maximum number of doughnuts, Jacob would have to spend all the money on doughnuts and none on muffins. One doughnut costs 25¢, so the number he can buy for \$15 is $15 \div 0.25 = 60$.

d) Answers may vary, e.g., $0.75m + 0.25d = 15$; m represents number of muffins bought, d represents number of doughnuts bought.

e)



3. Answers may vary, e.g., I think the table of values is most useful because it clearly lists some of Jacob's options.

4. Choose a value for one variable and then determine what value for the other variable makes the relation true. Answers may vary, e.g.,

a) For $x = 0$, $y = 5(0) - 1 = -1$, so (0, –1) satisfies the relation. For $x = 1$, $y = 5(1) - 1 = 4$, so (1, 4) satisfies the relation. For $x = 2$, $y = 5(2) - 1 = 9$, so (2, 5) does not satisfy the relation.

b) To make calculations easier, isolate y:

$$3x - 4y = 24$$

$$-4y = -3x + 24$$

$$y = \frac{3}{4}x - 6$$

For $x = 8$,

$$y = \frac{3}{4}(8) - 6$$

$$y = 6 - 6$$

$$y = 0$$

So (8, 0) satisfies the relation.

For $x = 0$,

$$y = \frac{3}{4}(0) - 6$$

$$y = 0 - 6$$

$$y = -6$$

So (0, –6) satisfies the relation.

For $x = 10$,

$$y = \frac{3}{4}(10) - 6$$

$$y = 7.5 - 6$$

$$y = 1.5$$

So (10, 2) does not satisfy the relation.

c) For $x = 0$, $y = -25(0) + 10 = 10$, so (0, 10) satisfies the relation. For $x = 2$, $y = -25(2) + 10 = -40$, so (2, –40) satisfies the relation and (2, –50) does not.

d) To make calculations easier, isolate y,

$5x = 30 - 2y$

$2y = 30 - 5x$

$$y = 15 - \frac{5}{2}x$$

For $x = 2$,

$$y = 15 - \frac{5}{2}(2)$$

$y = 15 - 5$

$y = 10$

So (2, 10) satisfies the relation.

For $x = 6$,

$$y = 15 - \frac{5}{2}(6)$$

$y = 15 - 15$

$y = 0$

So (6, 0) satisfies the relation and (0, 0) does not.

5. a) Let x represent the number of hours at the day job per week and let y represent the number of hours at the night job per week. Caroline works 40 h in all, so $x + y = 40$.

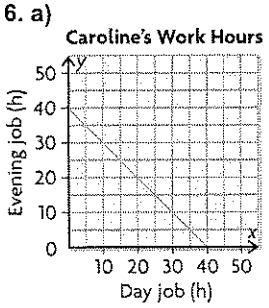
b) Let x represent the number of hours at the day job per week and let y represent the number of hours at the night job per week. Caroline earns \$15/h at the day job and \$11/h at her night job. Also, she earns \$540 per week. So, $15x + 11y = 540$.

c) Let x represent sales per week and let y represent earnings per week. Justin earns \$500 plus 6% of his sales, so $y = 500 + 0.06x$.

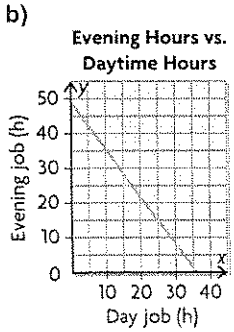
d) Let x represent sales per week and let y represent earnings per week. Justin would earn \$800, plus 4% of his sales, so $y = 800 + 0.04x$.

e) Let x represent number of nickels and let y represent number of dimes. Each nickel is worth \$0.05 and each dime is worth \$0.10, so $0.05x + 0.10y = 5.25$.

6. a)



b)



7. Evaluate each equation in parts c) and d) for $x = 18\,000$:

For the current job:

$y = 500 + 0.06x$

$= 500 + 0.06(18\,000)$

$= 500 + 1080$

$= 1580$

For the other job:

$y = 800 + 0.04x$

$= 800 + 0.04(18\,000)$

$= 800 + 720$

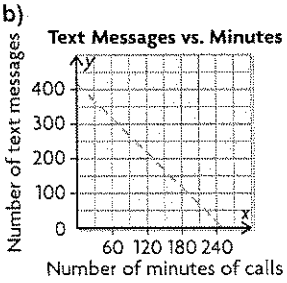
$= 1520$

Justin earns \$1580 per week in his current job and would earn only \$1520 per week at the new job. So, he should not take it.

8. a) Answers may vary, e.g.,

Phone Calls		Text Messages		Total
Min	Cost ($)	Number	Cost ($)	Cost ($)
250	25.00	0	0.00	25.00
190	19.00	100	6.00	25.00
130	13.00	200	12.00	25.00
70	7.00	300	18.00	25.00
10	1.00	400	24.00	25.00
0	0.00	416	24.96	24.96

b)



9. a) Let x represent sales and let y represent earnings. The earnings are \$1200 plus 3.5% of sales, so $y = 1200 + 0.035x$.

b) Substitute $x = 96\,174$ into the equation.

$y = 1200 + 0.035x$

$= 1200 + 0.035(96\,174)$

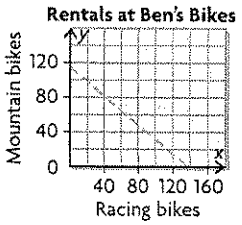
$= 1200 + 3366.09$

$= 4566.09$

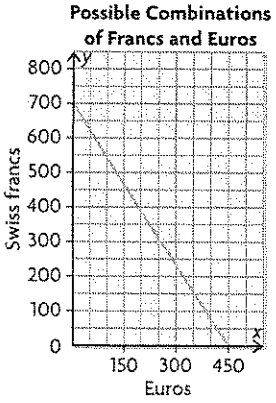
So, yes, the amount is correct.

10. a) The greatest number of racing bikes that could have been rented is when no mountain bikes were rented. The rental charges were \$3450, so the number of racing bikes rented would be $3450 \div 25 = 138$ bikes.

b) The greatest number of mountain bikes that could have been rented is when no racing bikes were rented. The rental charges were \$3450, so the number of mountain bikes rented would be $3450 \div 30 = 115$ bikes.
c) Let x represent the number of racing bikes rented and let y represent the number of mountain bikes rented;
$25x + 30y = 3450$



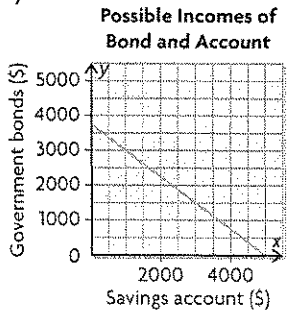
11. Answers may vary, e.g. let x represent euros and let y represent Swiss francs. One euro is \$1.40 and one Swiss franc is \$0.90, and they must equal \$630, so $1.40x + 0.90y = 630$. All the possible combinations are solutions to the equation. These can be shown in a graph



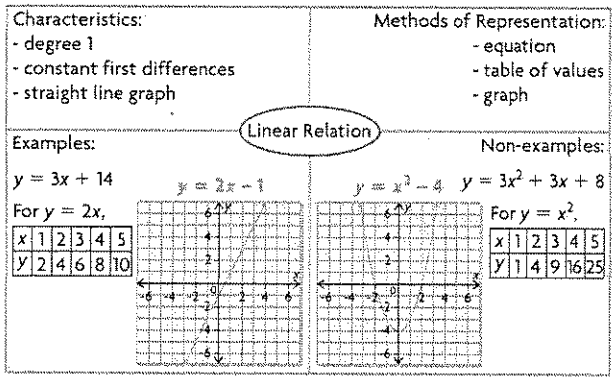
or a table of values

Euros (x)	Swiss Francs (y)
0	700
90	560
180	420
270	280
360	140
450	0

12. a) Let x represent the amount invested in the savings account and let y represent the amount invested in the government bond. The savings account earns 3% interest, the bond earns 4%, and the total earned is \$150, so $0.03x + 0.04y = 150$.
b)



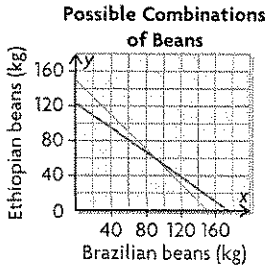
13. Let x represent the registration fee and let y represent the monthly fee. The total fee is \$775 and there are five months, so an equation to represent this relation is $x + 5y = 775$. Many combinations are possible; one is a registration fee of \$425 and a monthly fee of \$70 (425, 70).
14. a) Answers may vary, e.g.,



b) Answers may vary, e.g.,

Method:	Advantages:	Disadvantages:	Situations To Use:
equation	- can be used to determine any values of x or y	- requires arithmetic to determine answer	- when an exact number (such as one with several decimal places) is needed
table of values	- shows possible combinations at a glance	- does not list all values - may be difficult to see a pattern	- when several possible combinations are needed quickly
graph	- shows information visually	- can be hard to determine exact values	- when showing intercepts of two different relations

15. a) Answers may vary, e.g., you have \$4.65 in dimes and quarters.
b) Answers may vary, e.g., you are paid a base salary of \$900 and a 2.5% commission on sales.
16. Let x represent the amount of Brazilian beans and let y represent the amount of Ethiopian beans:
i) $x + y = 150$
ii) $12x + 17y = 14 \times 150$



Lesson 1.2: Solving Linear Equations, page 18

1. a) To estimate the rental cost, I determine 500 on the x-axis and draw a vertical line until I reach the line. I then read off the y-value which corresponds to the point. In this case, it is about \$125.

b) To estimate the driving distance, I determine the dollar amount on the y-axis and move horizontally until I reach the line. I then move down vertically and read the number of km off the x-axis. For \$80, I can drive about 200 km; for \$100, about 350 km; and for \$75, about 150 km.

2. a) Let C represent the cost and let d represent the distance. The y-intercept is 50, and to determine the slope I will use rise over run. My run is going from 0 to 200 and the rise is 30.

$$\text{slope} = \frac{30}{200}$$
$$= \frac{3}{20}$$

Using the $y = mx + b$ form, the equation of the graph is

$$C = 50 + \frac{3}{20}d\ .$$

b) i) When $d = 500$ then:

$$C = \frac{3}{20}d + 50$$
$$C = \frac{3}{20}(500) + 50$$
$$C = 75 + 50$$
$$C = 125$$

It would cost \$125 to travel 500 km.

ii) When $C = 80$:

$$C = \frac{3}{20}d + 50$$
$$80 = \frac{3}{20}d + 50$$
$$30 = \frac{3}{20}d$$
$$d = 200$$

When $C = 100$:

$$C = \frac{3}{20}d + 50$$
$$100 = \frac{3}{20}d + 50$$
$$50 = \frac{3}{20}d$$
$$d \quad 333.33$$

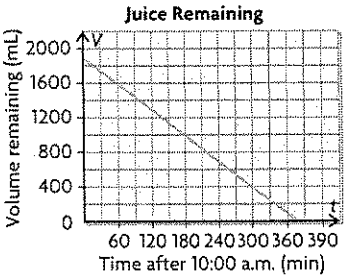
When $C = 75$:

$$C = \frac{3}{20}d + 50$$
$$75 = \frac{3}{20}d + 50$$
$$25 = \frac{3}{20}d$$
$$d \quad 166.67$$

The answers are 200 km; about 333.33 km; about 166.67 km respectively.

c) Using an equation gave more accurate answers.

3. a) Let t represent time in minutes after 10:00 a.m. and let V represent volume in millilitres remaining in the container. The y-intercept is 1890 and the slope is –5, since the rate of change is decreasing. The equation is $V = 1890 - 5t$.



b) To determine the time when there is 1 L remaining, I will let $V = 1$ and solve for t.

$$V = 1890 - 5t$$
$$1000 = 1890 - 5t$$
$$-890 = -5t$$
$$t = 178$$

There is 1 L of juice after 178 min, which is 12:58 p.m.

4. a) The point tells me that a banquet for 160 people will cost \$5700 at this hall.

b) To determine the charge for 200 people, I start at 200 on the x-axis and move up vertically until I touch the line. I then read off the y-value. A banquet for 200 would cost about \$7000.

c) Let C represent cost and let n represent number of people. The y-intercept is 500 and as the number of people increases from 0 to 50 the cost increases \$1625.

$$\text{slope} = \frac{1625}{50}$$
$$= 32.5$$

The equation of the graph is $C = 32.5n + 500$.

d) When $C = 3100$:

$$C = 32.5n + 500$$
$$3100 = 32.5n + 500$$
$$2600 = 32.5n$$
$$n = 80$$

When $C = 4400$:

$$C = 32.5n + 500$$
$$4400 = 32.5n + 500$$
$$3900 = 32.5n$$
$$n = 120$$

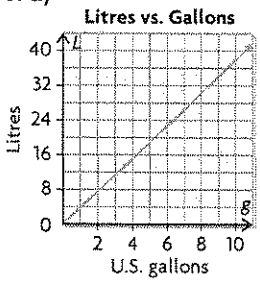
When $C = 5000$:

$$C = 32.5n + 500$$
$$5000 = 32.5n + 500$$
$$4500 = 32.5n$$
$$n \doteq 138$$

For the given prices I could invite 80, 120, and 138 people respectively.

e) A broken line is used for this graph because the data is discrete: the x-value must be an integer. You cannot have a fraction of a person attending a banquet.

5. a)



b) To estimate the number of litres, I start at 6 on the *x*-axis, I move up until I reach the graph and then I move horizontally towards the *y*-axis. I then read off the *y*-value. I determine that 6 gallons is about 23 L.
c) To estimate the number of gallons, I start at 14 on the *y*-axis, I move across until I reach the graph and then I move vertically towards the *x*-axis. I then read off the *x*-value. I determine that 14 L is about 3.5 gallons.
6. Since Melanie started with 35 L and the light came on when there were 9 L left, the light came on after 26 L had been used. To determine the distance she travelled, divide the gas used by the rate at which it is consumed.
$26 \div 0.095 \doteq 273.7$
So Melanie drove 273.7 km before the light came on. Since she was driving at 100 km/h, she drove for 2.73 h, which is 2 h and 44 min. The light came on at 4:59 p.m.
7. a) To determine the portion of Hank's required earnings which must come from commission, I subtract the amount he always earns. $900 - 280 = 620$
Hank needs to earn $620 through commission. To determine the required sales, I divide 620 by 0.04, his rate of commission. $620 \div 0.04 = 15\,500$
Hank needs to sell 15 500 dollars' worth of merchandise in order to make $900 per week.
b) Let E represent earnings and let s represent sales. Since Hank always earns $280, it is the constant term. The rate of change is 0.04, since this is his commission rate. The equation is $E = 280 + 0.04s$. When $E = 900$ then:

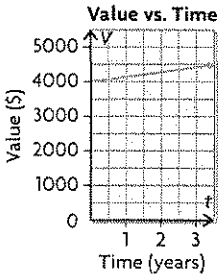
$$\begin{aligned} 900 &= 280 + 0.04s \\ 620 &= 0.04s \\ s &= 15\,500 \end{aligned}$$

8. a) The company charges $40 plus $10 per square foot. I can represent this relation as follows:
$C = 40 + 10t$. If Andrew has $3500, then I solve for t when $C = 3500$.

$$\begin{aligned} 3500 &= 40 + 10t \\ 3460 &= 10t \\ 346 &= t \end{aligned}$$

Andrew could have 346 square feet of paving stones installed.
b) Andrew will need to spend $5 \times 15 = \$75$ on sand. This will mean he has $75 less to spend on paving. Since paving stones cost $10/square foot, he will have to decrease his area by $75 \div 10 = 7.5$ square feet.

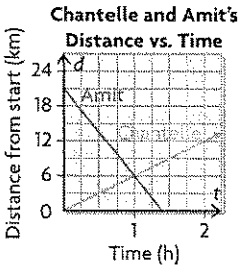
9. a) Let V represent the value and t represent time in years. The value equals the investment plus the interest earned times the number of years. The equation is $V = 4000 + (4000)(0.035)(t)$.



b) Draw a vertical line from 2 on the *x*-axis to the graph, then move horizontally to the *y*-axis and read the value, which is about $4280.
c) Draw a horizontal line from 4385 on the *y*-axis to the graph, then move vertically to the *x*-axis and read the value, which is about 2.75 years or 2 years 9 months.
10. The cost is $65 plus $0.12/km. The equation for this is $C = 65 + 0.12d$, where C is the cost and d is the distance. When $C = 90$:

$$\begin{aligned} C &= 65 + 0.12d \\ 90 &= 65 + 0.12d \\ 25 &= 0.12d \\ d &\doteq 208.33 \end{aligned}$$

Maria can drive about 208 km with her grandmother.
11. a) I could write equations, make a table of values or draw a graph.
b) For the first job to be Cam's choice, he would have to make at least $300 in commission. To determine what that means in sales, I divide the commission required by the commission rate: $300 \div 0.025 = 12\,000$. If Cam makes more than $12 000 of sales per week, he should stay at his first job as he will earn more money there. Otherwise, he should take the new job.
12.



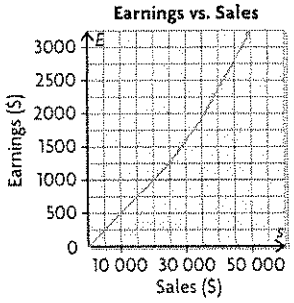
They will meet after 1 h, at 10:00 a.m.
13. Graphically, I would look along the horizontal line representing $y = 5$ until it intersected the line representing $2x - 3y = 6$. Algebraically, I would substitute $y = 5$ into the equation and solve for x.

14. Had Luke missed every time, he would have paid 0.25 × 25 = 6.25. That is close to 5.25, so I made a table of possibilities to determine what combination gives $5.25.

Number of misses	Money paid ($)	Number of hits	Money won ($)	Total money paid ($)
25	6.25	0	0	6.25
24	6.00	1	0.75	5.25

Luke hit the bull's-eye once.

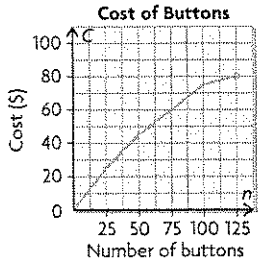
15.



I start at $2000 on the *y*-axis, draw a horizontal line across to the graph and read the corresponding *y*-coordinate. She needs to sell about $38 333.

16. a)

Number of buttons	Cost ($)
0	0
25	25
50	45
75	60
100	75
125	80



b) Answers may vary, e.g.,100 buttons cost $75 and 101 buttons cost $75.20. I would tell people to buy several extra now, while they are only $0.20 each, so that if they needed more later, they wouldn't have to buy them at $1.00 each. This pricing structure encourages people to buy more buttons since the more they buy, the cheaper the price is per button.

c) Each price range needs its own equation since it has a different rate of change. Let *C* represent the cost in dollars and let *n* represent the number of buttons. Since the price is $1 per button, for *n* from 1 to 25: $C = n$. The first 25 buttons cost $25; this is the fixed cost. Every button after 25 $(n - 25)$ costs $0.80. So, for *n* from 26 to 50: $C = 25 + 0.8(n - 25)$.

The first 50 buttons cost 25 + 0.8(25) = $45; this is the fixed cost. Every button after 50 $(n - 50)$ costs $0.60. For *n* from 51 to 100: $C = 45 + 0.6(n - 50)$. The first 100 buttons cost 25 + 0.8(25) +0.6(50) = $75; this is the fixed cost. Every button after $100(n - 100)$ costs $0.20.

For *n* from 101 and higher:
$C = 75 + 0.2(n - 100)$.

Lesson 1.3 Graphically Solving Linear Systems, page 26

1. a) Equation 1:

$$\begin{aligned} LS &= 3x + 2y \\ &= 3(2) + 2(-1) \\ &= 6 - 2 \\ &= 4 \\ &= RS \end{aligned}$$

Equation 2:

$$\begin{aligned} LS &= -x + 3y \\ &= -(2) + 3(-1) \\ &= -5 \\ &= RS \end{aligned}$$

Yes, (2, –1) is a solution.

b) Equation 1:

$$\begin{aligned} LS &= x + y \\ &= (1) + (4) \\ &= 5 \\ &= RS \end{aligned}$$

Equation 2:

$$\begin{aligned} LS &= 2x + 2y \\ &= 2(1) + 2(4) \\ &= 2 + 8 \\ &= 10 \\ &\neq RS \end{aligned}$$

No, (1, 4) is not a solution.

c) Equation 1:

$$\begin{aligned} LS &= 3x - 5 \\ &= 3(1) - 5 \\ &= -2 \\ &= y \\ &= RS \end{aligned}$$

Equation 2:

$$\begin{aligned} LS &= 2x - 4 \\ &= 2(1) - 4 \\ &= -2 \\ &= y \\ &= RS \end{aligned}$$

Yes, (1, –2) is a solution.

d) Equation 1:

$$\begin{aligned} LS &= x - y \\ &= 10 - 5 \\ &= 5 \\ &= RS \end{aligned}$$

Equation 2:

$$\begin{aligned} LS &= 5x - 40 \\ &= 5(10) - 40 \\ &= 10 \\ &\neq RS \end{aligned}$$

No, (10, 5) is not a solution.

2. a) i) (–4, –2)

ii) Equation 1:

LS $= 3x + y$
$= 3(-4) + (-2)$
$= -12 - 2$
$= -14$
$=$ RS

Equation 2:

RS $= 2y$
$= 2(-2)$
$= -4$
$= x$
$=$ LS

Yes, (–4, –2) is a solution.

b) i) (6, 5)

ii) Equation 1:

LS $= x + 3y$
$= (6) + 3(5)$
$= 6 + 15$
$= 21$
$=$ RS

Equation 2:

LS $= x - y$
$= (6) - (5)$
$= 1$
$=$ RS

Yes, (6, 5) is a solution.

c) i) (5, –4)

ii) Equation 1:

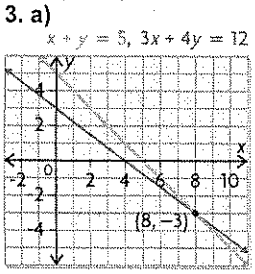
LS $= x + y$
$= (5) + (-4)$
$= 1$
$=$ RS

Equation 2:

LS $= 2x - 5y$
$= 2(5) - 5(-4)$
$= 10 + 20$
$= 30$
$=$ RS

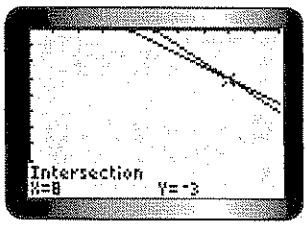
Yes, (5, –4) is a solution.

3. a)



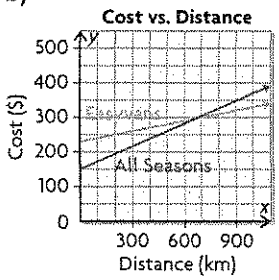
b) The point where the two lines intersect is (8, –3).

c)



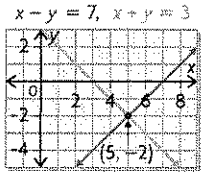
4. a) Let x represent the distance driven and let y represent the total cost in dollars. For Easyvans, there is a fixed cost of \$230 and a rate of change of \$0.10. The equation is $y = 230 + 0.10x$. For Cars for All Seasons, there is a fixed cost of \$150 and a rate of change of \$0.22. The equation is $y = 150 + 0.22x$.

b)

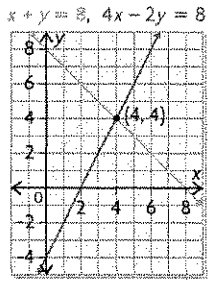


c) I would recommend Easyvans if Alex was planning on driving more than 667km, because it would be cheaper. Otherwise, I would recommend Cars for All Seasons because it would be cheaper.

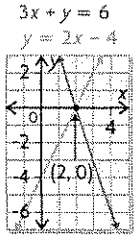
5. a) The point of intersection is (5, –2).



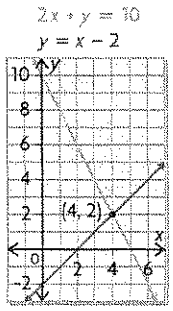
b) The point of intersection is (4, 4).



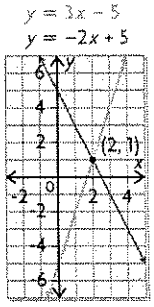
c) The point of intersection is (2, 0).



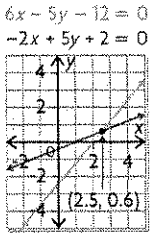
d) The point of intersection is (4, 2).



e) The point of intersection is (2, 1).



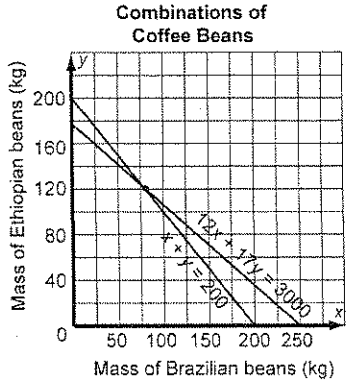
f) The point of intersection is (2.5, 0.6).



6. a) The red line has a slope of –1 and a *y*-intercept of 80. The equation is $y = -x + 80$ or $x + y = 80$. The blue line has a *y*-intercept of 100 and a slope of –1.8. I found the slope by using rise over run: when I started at 100 and dropped down to 10, the run was 50. The equation is $y = -1.8x + 100$ or $18x + 10y = 1000$.

b) The amount of each blend is shown by the point of intersection on the graph. He will use 25 kg of pineapple mix and 55 kg of banana mix.

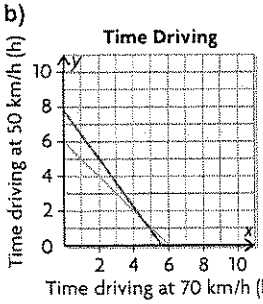
7. Let *x* represent the number of kilograms of Brazilian beans and represent the number of kilograms of Ethiopian beans. The amount of both beans added together must equal 200 kg, therefore $x + y = 200$. A second equation will be the prices multiplied by their quantity, $12x + 17y = 3000$. Solve by graphing these two equations.



From the graph, the point of intersection is (80, 120). So, Jessica should use 80 kg of the Brazilian beans and 120 kg of the Ethiopian beans.

8. a) Let *x* represent time driving at 70 km/h and let *y* represent time driving at 50 km/h. The total time is 6 hours, so an equation to represent the total distance is $x + y = 6$. Since distance = speed × time, the equation for the total distance is $70x + 50y = 393$.

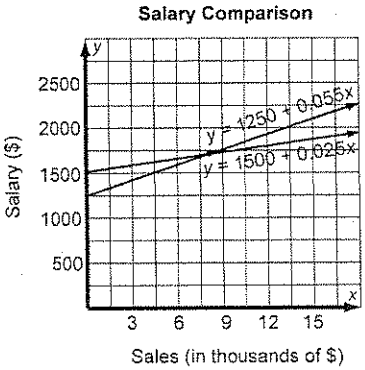
b)



c) From the graph, the lines intersect at (4.65, 1.35). So he spent 4.65 h driving at 70 km/h and 1.35 h driving at 50 km/h. 1.35 h at 50km/h is 67.5 km, so there is 67.5 km between the towns.

9. a) Let *y* represent earnings and let *x* represent sales. Phoenix has a fixed salary of 1500 and a rate of change of 0.025. The equation is $y = 1500 + 0.025x$ for Phoenix. Joanna has a fixed salary of 1250 and a rate of change of 0.055. The equation is $y = 1250 + 0.055x$ for Styles by Rebecca.

b) Solve by graphing the equations.

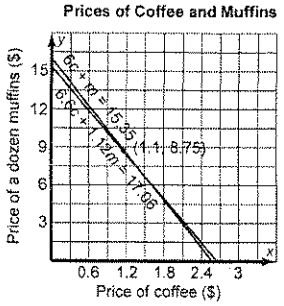


The lines intersect at about (8333, 1708), so, about $8333 of sales would result in the same salary.

c) From the graph, Phoenix Fashions has a lower salary until after about $8333 of sales. So if Joanna expects to have more than $8333 of sales, she should work at Phoenix Fashions. Otherwise, she should work at Styles by Rebecca because that job would pay her more.

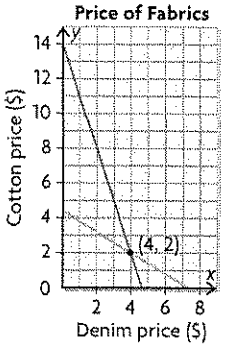
10. Answers may vary. E.g., At a fundraiser barbecue, hamburgers cost $3 and hotdogs cost $2. Martha is in charge of buying lunch for her class of 25 students. Each student wants either a hotdog or a hamburger. If she has collected $60 to spend, how many of each should she buy?

11. Let c represent the cost of a cup of coffee and let m represent the cost of a dozen muffins. Then $6c + m = \$15.35$ represents the cost of 6 coffees and a dozen muffins. The equation $6(1.10)c + 1.12m = \$17.06$, or $6.6c + 1.12m = \$17.06$, represents 6 coffees after a 10% increase and a 12% increase in the price of muffins. Solve by graphing the equations.



From the graph, the point of intersection is (1.1, 8.75), thus, the original price of a coffee was \$1.10, and the original price of a dozen muffins was \$8.75. The new price of a coffee and a dozen muffins is:
$1.1(1.1) + 1.12(8.75) = \$11.01$.

12. a) Let x represent the price of denim fabric per metre and let y represent the price of cotton fabric per metre. Willow bought 3 m of denim ($3x$) and 5 m of cotton fabric ($5y$), and it cost her \$22. The equation is $3x + 5y = 22$. Jared bought 6 m of denim ($6x$) and 2 m of cotton fabric ($2y$), and it cost him \$28. The equation is $6x + 2y = 28$.
b) The solution is (4, 2): denim costs \$4/m, cotton costs \$2/m.

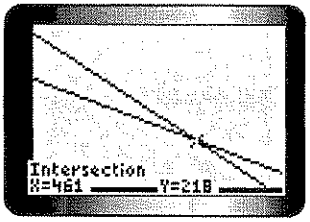


c) $8x + 5y = 8(4) + 5(2)$
$= 32 + 10$
$= 42$

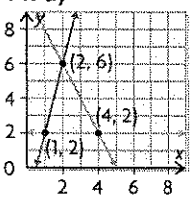
It will cost \$42 for 8 m of denim fabric and 5 m of cotton fabric.

13. a) Let x represent the number of student tickets and let y represent the number of non-student tickets. The total number of people equals 679, so $x + y = 679$. The revenue from students ($4x$) plus the revenue from non-students ($7y$) equals \$3370, so $4x + 7y = 3370$.

b) Since the y-coordinate equals the number of non-students, 218 non-students saw the play.



14. a)

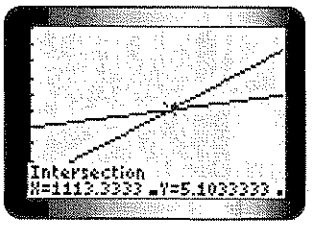


The coordinates of the vertices are (2, 6), (1, 2) and (4, 2).
b) The height of the triangle is represented in the y coordinates. Since they range from 2 to 6, the height is 4 units. The base is shown in the range of the x-coordinates. They range from 1 to 4, so the base is 3 units.

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(3)(4)$$
$$= 6$$

The area of the triangle is 6 square units.

15. a) Let C represent cost and let t represent the usage time. For a regular bulb, the fixed cost is \$0.65 and the rate of change is \$0.04. The equation is $C = 0.65 + 0.004t$. For a fluorescent bulb, the fixed cost is \$3.99 and the rate of change is \$0.01. The equation is $C = 3.99 + 0.001t$.
b)



c) The fluorescent light bulb is cheaper after about 1113 h.
d) Determine the number of hours in a year:
$365 \times 24 = 8760$
Now determine the cost for the regular light bulb:
$C = 0.65 + 0.004t$
$C = 0.65 + 0.004(8760)$
$C = 0.65 + 35.04$
$C = 35.69$

The cost for the fluorescent light bulb is:

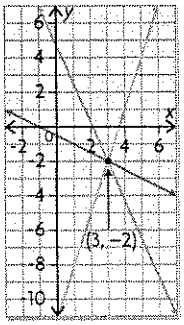
$C = 3.99 + 0.001t$
$C = 3.99 + 0.001(8760)$
$C = 3.99 + 8.76$
$C = 12.75$

The fluorescent bulb costs 35.69 – 12.75 = $22.94 less than the regular bulb.

16. Answers may vary, e.g.,

- Read the problem and determine what you need to figure out.
- Write two equations that describe the situation in the problem.
- Choose the best strategy to graph each equation.
- Label the graph.
- Graph both equations on the same set of axes.
- Determine the coordinates of the point of intersection.
- Verify the solution by substituting into both equations.

17. a) and b)



$3x - y - 11 = 0$
$x + 2y + 1 = 0$
$9x + 4y - 19 = 0$

All three lines pass through the point (3, –2).

c) $c(3x - y - 11) + d(x + 2y + 1) = 0$
$3cx - cy - 11c + dx + 2dy + d = 0$
$3cx + dx + 2dy - cy - 11c + d = 0$
$(3c + d)x + (2d - c)y + (d - 11c) = 0$

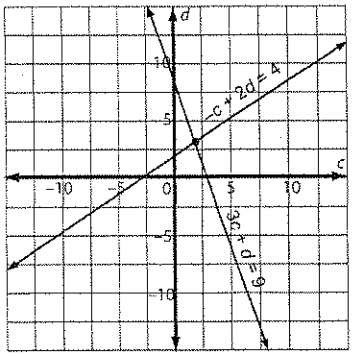
But, I know $9x + 4y - 19 = 0$. So, the coefficients of x in both equations must be equal:

$3c + d = 9$

Likewise, the coefficients of y must be equal:

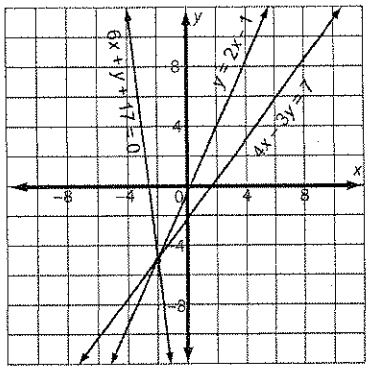
$-c + 2d = 4$

Solve by graphing these two equations.



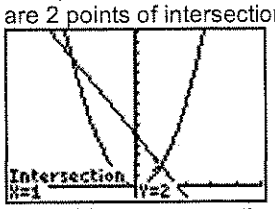
The two lines intersect at (2, 3), so $c = 2$ and $d = 3$.

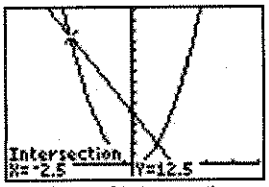
18.



The three lines intersect at (–2, –5). I did not have to graph all three lines; I could have graphed two and used the third to check the answer algebraically.

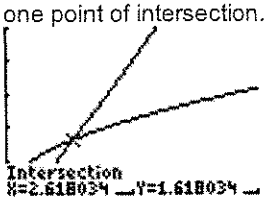
19. a) Using graphing technology, we can see that there are 2 points of intersection.





From this we can see that the points of intersection are (1, 2) and (–2.5, 12.5).

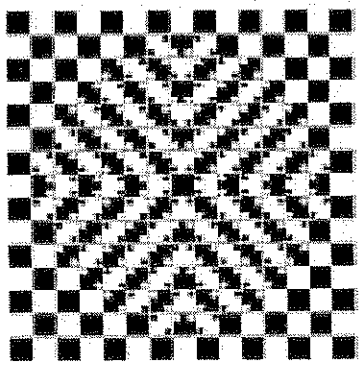
b) Using graphing technology, we can see that there is one point of intersection.



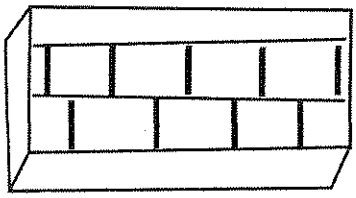
From this, we can see that the point of intersection is about (2.618, 1.618).

Curious Math, page 29

1. I found many optical illusions. Most of them make identical objects appear to be different sizes. There are also other illusions, which give the appearance of movement. This optical illusion makes straight lines appear to be curved.



2. In this diagram the thick black vertical lines appear to be different lengths. The lines appear longer when they intersect the horizontal lines and shorter when they do not. This happens since the horizontal lines are not parallel.

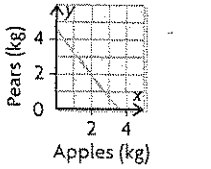


Mid-Chapter Review, page 32

1. a) Answers may vary, e.g.,

Apples		Pears		Total Cost ($)
Mass (kg)	Cost ($)	Mass (kg)	Cost ($)	
0.00	$0 \times 2.84 = 0.00$	4.58	$4.58 \times 2.18 = 9.98$	9.98
1.00	$1 \times 2.84 = 2.84$	3.28	$3.28 \times 2.18 = 7.15$	9.99
2.00	$2 \times 2.84 = 5.68$	1.98	$1.98 \times 2.18 = 4.32$	10.00
3.00	$3 \times 2.84 = 8.52$	0.67	$0.67 \times 2.18 = 1.46$	9.98
3.52	$3.52 \times 2.84 = 10.00$	0.00	$0 \times 2.18 = 0.00$	10.00

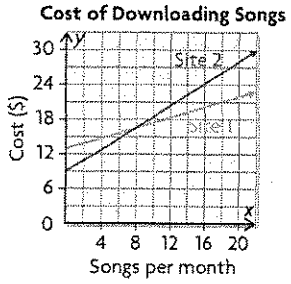
b)



c) Let x represent the mass of apples and let y represent the mass of pears. The cost of each fruit is multiplied by its weight and added together to get $10.00. The algebraic representation of this is $2.84x + 2.18y = 10.00$.

2.

Songs per month	Cost for Site 1 ($)	Cost for Site 2 ($)
0	12.95 + 0.45(0) = 12.95	8.99 + 0.95(0) = 8.99
5	12.95 + 0.45(5) = 15.20	8.99 + 0.95(5) = 13.74
10	12.95 + 0.45(10) = 17.45	8.99 + 0.95(10) = 18.49
15	12.95 + 0.45(15) = 19.70	8.99 + 0.95(15) = 23.24
20	12.95 + 0.45(20) = 21.95	8.99 + 0.95(20) = 27.99



3. a) The coordinates tell me that if she has $1500 in sales, she will earn $360.

b) Draw a line vertically from 3200 on the x-axis to the graph, then go across horizontally to the y-axis. The value on the y-axis is about $425. She would earn about $425.

c) Draw a line horizontally from 450 on the y-axis to the graph, then go down to the x-axis. The value on the x-axis is about $3700. She would need to sell about $3700 worth of merchandise.

d) Let y represent earnings and let x represent sales. Sally makes $300 as a fixed salary and makes 4% from commission, which is the rate of change. The equation representing Sally's salary is $y = 300 + 0.04x$.

For part b), when $x = 3200$,

$$\begin{aligned} y &= 300 + 0.04x \\ &= 300 + 0.04(3200) \\ &= 300 + 128 \\ &= 428 \end{aligned}$$

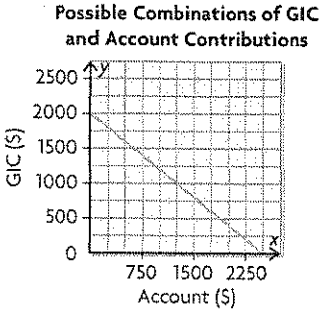
For part c), when $y = 450$,

$$\begin{aligned} y &= 300 + 0.04x \\ 450 &= 300 + 0.04x \\ 150 &= 0.04x \\ x &= 3750 \end{aligned}$$

4. a) Answers may vary, e.g., Let P represent the perimeter and let x represent the length. The length is x and the width is $x - 8$. The perimeter is $P = 2x + 2(x - 8)$.

b) If the width is 72 cm then $x - 8 = 72$, and $x = 80$ cm.

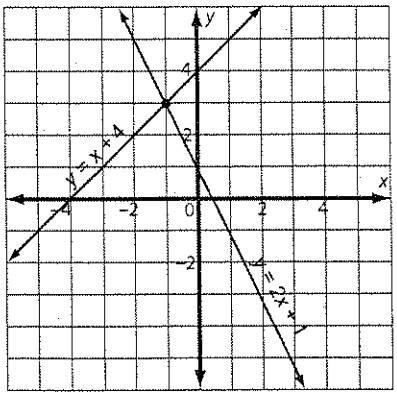
5. a) Let x represent the amount in the account and let y represent the amount in the GIC. To represent this information in an equation, I must multiply the amount of money in the account by 4% and the amount of money in the GIC by 5%, and then add them together. They equal $100 since that is the amount of interest he earned during the year. The equation is $0.04x + 0.05y = 100$.



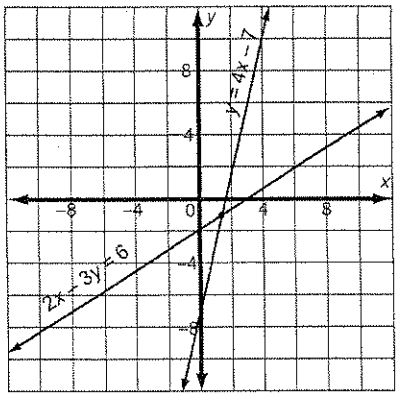
b) Draw a horizontal line from 1500 on the y-axis to the graph, then down to determine the x-value of that point, which is about $625. He needs to put $625 into his account.

c) Draw a vertical line from 2200 on the x-axis to the graph, then across to determine the y-value of that point, which is about \$240. He needs to put \$240 into a GIC.

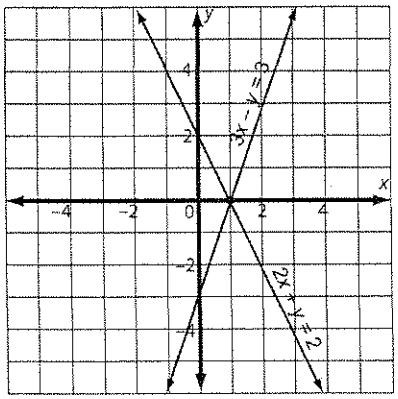
6. a) From the graph, the solution is $x = -1$ and $y = 3$.



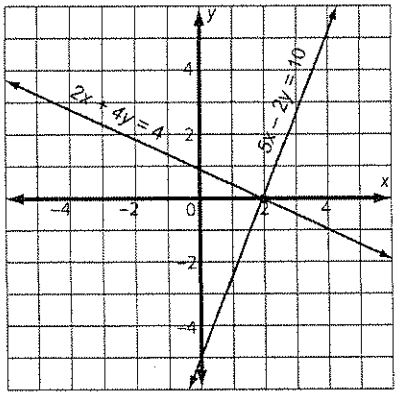
b) From the graph, the solution is $x = 1.5$ and $y = -1$.



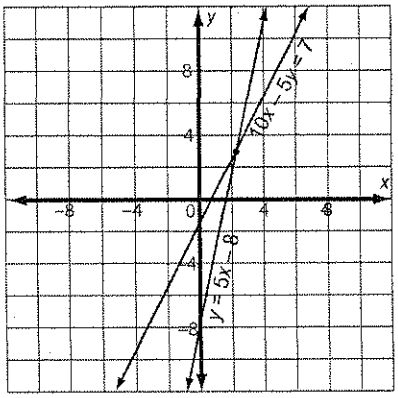
c) From the graph, the solution is $x = 1$ and $y = 0$.



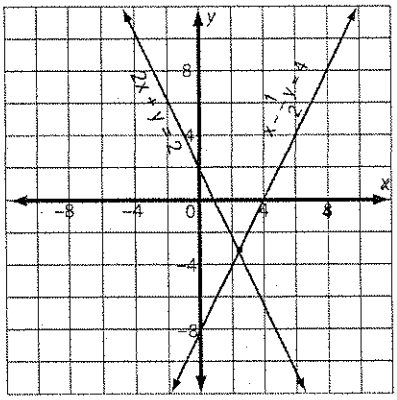
d) From the graph, the solution is $x = 2$ and $y = 0$.



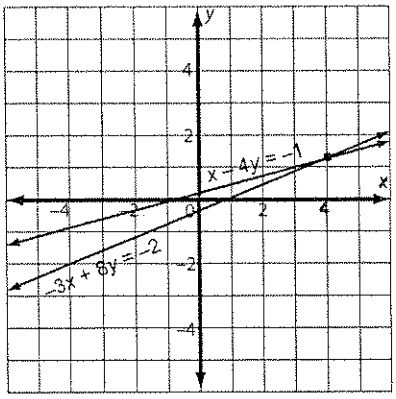
7. a) From the graph, the solution is $x = 2.2$ and $y = 3$.



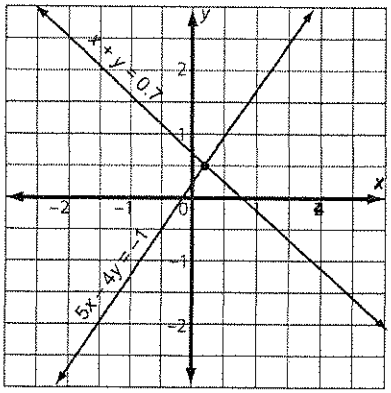
b) From the graph, the solution is $x = 2.5$ and $y = -3$.



c) From the graph, the solution is $x = 4$ and $y = 1.25$.

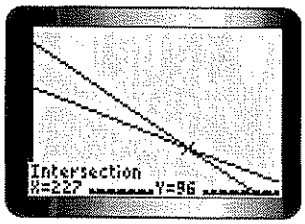


d) From the graph, the solution is $x = 0.2$ and $y = 0.5$.



8. a) Let x represent student tickets and let y represent non-student tickets. The total number of people is 323, so $x + y = 323$. I also know that the total amount made is \$790. To determine an algebraic representation of this, I multiply the type of ticket (student vs. non-student) by the price for each. This is represented algebraically by $2x + 3.50y = 790$.

b)



From the graph, the solution to this system is (227, 96). There were 227 students and 96 non-students in attendance.

9. Yes. For any ordered integer pair (x, y) there exist m and n that satisfy the equations. Therefore the solution could be in any quadrant. For instance, if I wanted to determine a solution in the 2nd quadrant, I could choose any point in that quadrant, e.g. $(-1, 4)$. This point is the solution to my system. Now I substitute the point into both equations and determine the required m and n values. First, I solve for m:

$$\begin{aligned} y &= 2x + m \\ 4 &= 2(-1) + m \\ 6 &= m \end{aligned}$$

Now I solve for n:

$$\begin{aligned} n &= 3x - y \\ &= 3(-1) - 4 \\ &= -7 \end{aligned}$$

I could do this for each quadrant and each point; therefore, there is a solution in every quadrant.

10. Answers may vary, e.g., Dan and Heidi are playing table tennis against each other. After 17 points, Heidi is ahead by 7. How many points has each player scored?

Lesson 1.4: Solving Linear Systems: Substitution, page 38

1. a)

$$\begin{aligned} 10x - y &= 1 \\ y &= 10x - 1 \end{aligned}$$

b)

$$4x - y + 3 = 0$$

$$4x = y - 3$$

$$x = \frac{1}{4}y - \frac{3}{4}$$

c)

$$\frac{1}{2}x + y = 10$$

$$\frac{1}{2}x = 10 - y$$

$$x = 20 - 2y$$

d)

$$\begin{aligned} 2x - y &= 12 \\ y &= 2x - 12 \end{aligned}$$

2. Let x represent the number of cars and let y represent the number of vans.

a) To determine the number of vehicles washed, I add the number of vans to the number of cars. The total is 53: $x + y = 53$

b) To determine the amount of money made, I multiply the type of vehicle by the price charged and then add them together. The total is \$382: $6x + 8y = 382$

c) Answers may vary, e.g.,

$$\begin{aligned} x + y &= 53 \\ x &= 53 - y \end{aligned}$$

d) Answers may vary, e.g.,

$$\begin{aligned} 6(53 - y) + 8y &= 382 \\ 318 - 6y + 8y &= 382 \\ 2y &= 64 \\ y &= 32 \end{aligned}$$

e) Answers may vary, e.g.,

$$\begin{aligned} x + y &= 53 \\ x + 32 &= 53 \\ x &= 21 \end{aligned}$$

The students washed 21 cars and 32 vans.

3. a) Isolate x:

$$\begin{aligned} x + 3y &= 5 \\ x &= 5 - 3y \end{aligned}$$

Substitute:

$$\begin{aligned} 2(5 - 3y) - 3y &= -17 \\ 10 - 6y - 3y &= -17 \\ 10 - 9y &= -17 \\ -9y &= -27 \\ y &= 3 \end{aligned}$$

Solve for x:

$$\begin{aligned} x &= 5 - 3y \\ &= 5 - 3(3) \\ &= 5 - 9 \\ &= -4 \end{aligned}$$

The solution is $(-4, 3)$.

b) Isolate y:

$$\begin{aligned} 2x + y &= 4 \\ y &= 4 - 2x \end{aligned}$$

Substitute:

$$\begin{aligned} 3x - 16y &= 6 \\ 3x - 16(4 - 2x) &= 6 \\ 3x - 64 + 32x &= 6 \\ 35x &= 70 \\ x &= 2 \end{aligned}$$

Solve for y:

$$\begin{aligned} y &= 4 - 2x \\ &= 4 - 2(2) \\ &= 4 - 4 \\ &= 0 \end{aligned}$$

The solution is (2, 0).

4. a) $8a = 4 - b$

$b + 8a = 4$

$b = 4 - 8a$

b) $6r + 3s = 9$

$6r = 9 - 3s$

$r = \frac{3}{2} - \frac{1}{2}s$

c) $3u + 7v = 21$

$7v = 21 - 3u$

$v = 3 - \frac{3}{7}u$

d)

$0.3x - 0.3y = 1.8$

$0.3x = 1.8 + 0.3y$

$x = 6 + y$

e) $0.12x - 0.06y = 0.24$

$0.12x - 0.24 = 0.06y$

$\frac{0.12x}{0.06} - \frac{0.24}{0.06} = \frac{0.06y}{0.06}$

$2x - 4 = y$

$y = 2x - 4$

f) $\frac{1}{3}x + \frac{1}{2}y = 5$

$\frac{1}{3}x = 5 - \frac{1}{2}y$

$x = 15 - \frac{3}{2}y$

5. a) Isolate: $y = x - 5$

Substitute:

$$\begin{aligned} x + (x - 5) &= 9 \\ 2x - 5 &= 9 \\ 2x &= 14 \\ x &= 7 \end{aligned}$$

Solve for y:

$$\begin{aligned} y &= x - 5 \\ y &= 7 - 5 \\ y &= 2 \end{aligned}$$

The solution is (7, 2).

b) Isolate: $x = y + 4$

Substitute:

$$\begin{aligned} 3(y + 4) + y &= 16 \\ 3y + 12 + y &= 16 \\ 4y &= 4 \\ y &= 1 \end{aligned}$$

Solve for x:

$$\begin{aligned} x &= y + 4 \\ x &= 1 + 4 \\ x &= 5 \end{aligned}$$

The solution is (5, 1).

c) Isolate: $y = 4x - 16$

Substitute:

$$\begin{aligned} x + 4(4x - 16) &= 21 \\ x + 16x - 64 &= 21 \\ 17x &= 85 \\ x &= 5 \end{aligned}$$

Solve for y:

$$\begin{aligned} y &= 4x - 16 \\ y &= 4(5) - 16 \\ y &= 20 - 16 \\ y &= 4 \end{aligned}$$

The solution is (5, 4).

d) Isolate:

$$\begin{aligned} x + 3y &= 7 \\ x &= 7 - 3y \end{aligned}$$

Substitute:

$$\begin{aligned} 3x - 2y &= 10 \\ 3(7 - 3y) - 2y &= 10 \\ 21 - 9y - 2y &= 10 \\ -11y &= -11 \\ y &= 1 \end{aligned}$$

Solve for x:

$$\begin{aligned} x &= 7 - 3y \\ x &= 7 - 3(1) \\ x &= 7 - 3 \\ x &= 4 \end{aligned}$$

The solution is (4, 1).

e) Isolate:

$$\begin{aligned} 2x + y &= 5 \\ y &= 5 - 2x \end{aligned}$$

Substitute:

$$\begin{aligned} x - 3(5 - 2x) &= 13 \\ x - 15 + 6x &= 13 \\ 7x &= 28 \\ x &= 4 \end{aligned}$$

Solve for y:

$$\begin{aligned} y &= 5 - 2x \\ y &= 5 - 2(4) \\ y &= 5 - 8 \\ y &= -3 \end{aligned}$$

The solution is (4, −3).

f) Isolate:

$$\begin{aligned} x + 2y &= 0 \\ x &= -2y \end{aligned}$$

Substitute:

$x - y = -4$

$(-2y) - y = -4$

$-3y = -4$

$y = \frac{4}{3}$

Solve for x:

$x = -2y$

$x = -2\left(\frac{4}{3}\right)$

$x = -\frac{8}{3}$

The solution is $\left(-\frac{8}{3}, \frac{4}{3}\right)$.

6. Let the registration fee be x and the monthly charge be y. Since there are two variables, I will use a system of equations. The equation to represent the amount paid after 4 months is $x + 4y = 420$; the amount paid after 9 months is $x + 9y = 795$. Isolate x in the first equation.

$x = 420 - 4y$

Substitute the first equation into the second equation.

$$\begin{aligned} x + 9y &= 795 \\ (420 - 4y) + 9y &= 795 \\ 420 + 5y &= 795 \\ 5y &= 375 \\ y &= 75 \end{aligned}$$

Solve for x:

$x = 420 - 4y$
$x = 420 - 4(75)$
$x = 420 - 300$
$x = 120$

The registration fee is \$120 and the monthly charge is \$75.

7. Let x represent the number of 250 g jars and y represent the number of 500 g jars. Since I have two variables, I will use a system of equations. The number of jars is equal to the sum of both kinds of jars, $x + y = 511$. The total mass is the number of each type of jar multiplied by the mass it can holds. Using kilograms for the relation, I get $0.250x + 0.500y = 186.5$. Now I isolate x in the first equation and substitute it into the second.

$x = 511 - y$

$$\begin{aligned} 0.250x + 0.500y &= 186.5 \\ 0.250(511 - y) + 0.500y &= 186.5 \\ 127.75 - 0.250y + 0.500y &= 186.5 \\ 0.250y &= 58.75 \\ y &= 235 \end{aligned}$$

Since $y = 235$, I can solve for x:

$x = 511 - y$
$x = 511 - 235$
$x = 276$

The number of 500 g jars is 235 and the number of 250 g jars is 276.

8. Let x and y each represent an angle. If the difference between the angles is 11° then $x - y = 11$. If the sum of the two angles is 77° then $x + y = 77$. Since I have two equations, I can determine the angles.

I isolate x in the first equation: $x = y + 11$

I then substitute this new equation into the second equation:

$$\begin{aligned} (y + 11) + y &= 77 \\ 2y &= 66 \\ y &= 33 \end{aligned}$$

I solve for x:

$x = y + 11$
$x = 33 + 11$
$x = 44$

Since I know the angles of a triangle add to 180°, I can determine the third angle:

$z = 180° - 33° - 44°$
$z = 103°$

The angles are 33°, 44° and 103°.

9. Answers can be checked using a left-side-right-side check.

a) Isolate:

$$\begin{aligned} x + 3y &= 7 \\ x &= 7 - 3y \end{aligned}$$

Substitute:

$$\begin{aligned} 3x - 2y &= -12 \\ 3(7 - 3y) - 2y &= -12 \\ 21 - 9y - 2y &= -12 \\ -11y &= -33 \\ y &= 3 \end{aligned}$$

Solve for x:

$x = 7 - 3y$
$x = 7 - 3(3)$
$x = -2$

The solution is $x = -2$, $y = 3$.

b) Isolate:

$$\begin{aligned} 3 &= 2a - b \\ 3 + b &= 2a \\ b &= 2a - 3 \end{aligned}$$

Substitute:

$$\begin{aligned} 4a - 3b &= 5 \\ 4a - 3(2a - 3) &= 5 \\ 4a - 6a + 9 &= 5 \\ -2a &= -4 \\ a &= 2 \end{aligned}$$

Solve for b:

$b = 2a - 3$
$b = 2(2) - 3$
$b = 1$

The solution is $a = 2$, $b = 1$.

c) Isolate:

$$\begin{aligned} 7m + 2n &= 21 \\ 2n &= 21 - 7m \\ n &= 10.5 - 3.5m \end{aligned}$$

Substitute:

$$\begin{aligned} 10m + 4n &= -10 \\ 10m + 4(10.5 - 3.5m) &= -10 \\ 10m + 42 - 14m &= -10 \\ -4m &= -52 \\ m &= 13 \end{aligned}$$

Solve for n:

$n = 10.5 - 3.5m$
$n = 10.5 - 3.5(13)$
$n = 10.5 - 45.5$
$n = -35$

The solution is $m = 13$, $n = -35$

d) Isolate:

$6x - 2y + 1 = 0$

$6x + 1 = 2y$

$3x + \frac{1}{2} = y$

Substitute:

$$3x - 5y + 7 = 0$$
$$3x - 5\left(3x + \frac{1}{2}\right) + 7 = 0$$
$$3x - 15x - \frac{5}{2} + 7 = 0$$
$$-12x + \frac{9}{2} = 0$$
$$\frac{9}{2} = 12x$$
$$x = \frac{3}{8}$$

Solve for y:

$$y = 3x + \frac{1}{2}$$
$$y = 3\left(\frac{3}{8}\right) + \frac{1}{2}$$
$$y = \frac{9}{8} + \frac{1}{2}$$
$$y = \frac{13}{8}$$

The solution is $x = \frac{3}{8}, y = \frac{13}{8}$

e) Isolate:

$$3c + 2d = -24$$
$$2d = -24 - 3c$$
$$d = -12 - 1.5c$$

Substitute:

$$2c + 5d = -38$$
$$2c + 5(-12 - 1.5c) = -38$$
$$2c - 60 - 7.5c = -38$$
$$-5.5c = 22$$
$$c = -4$$

Solve for d:

$d = -12 - 1.5c$
$d = -12 - 1.5(-4)$
$d = -12 + 6$
$d = -6$

The solution is $c = -4$, $d = -6$.

f) Isolate:

$$\frac{1}{4}x - 3y = \frac{1}{2}$$
$$\frac{1}{4}x = 3y + \frac{1}{2}$$
$$x = 12y + 2$$

Substitute:

$$\frac{1}{3}x - 9y = 5$$
$$\frac{1}{3}(12y + 2) - 9y = 5$$
$$4y + \frac{2}{3} - 9y = 5$$
$$-5y + \frac{2}{3} = 5$$
$$-5y = \frac{13}{3}$$
$$y = -\frac{13}{15}$$

Solve for x:

$$x = 12y + 2$$
$$x = 12\left(-\frac{13}{15}\right) + 2$$
$$x = -\frac{156}{15} + 2$$
$$x = -\frac{42}{5}$$

The solution is $x = -\frac{42}{5}, y = -\frac{13}{15}$.

10. Let x represent the number of cheques used and y represent the total amount charged. Save-A-Lot Trust can the represented by $y = 0.75x + 10$ and Maple Leaf Savings can be represented by $y = x + 7$. To determine the point at which Save-A-Lot becomes cheaper, I will solve this system of equations.

$$x + 7 = 0.75x + 10$$
$$0.25x = 3$$
$$x = 12$$

If Dan uses more than 12 cheques per month then Save-A-Lot Trust charges less; if he uses fewer than 12 cheques per month then Maple Leaf Savings charges less. If he uses exactly 12 cheques then both charge the same amount.

11. Let x be the number of grams of the 80% alloy and y be the number of grams of the 66% alloy. Together the two alloys must be 30.00 g, therefore $x + y = 30.00$. Also, the mixture of the two alloys must be 70% silver, therefore this equation must also be satisfied:
$0.8x + 0.66y = 0.7(30)$.
Isolate: $x = 30 - y$
Substitute and solve:

$$0.8(30 - y) + 0.66y = 30$$
$$24 - 0.8y + 0.66y = 21$$
$$-0.14y = -3$$
$$y = 21.43$$

Solve for x:

$$x = 30 - y$$
$$x = 30 - 21.43$$
$$x = 8.57$$

Wayne needs about 8.57 g of 80% silver and about 21.43 g of 66% silver.

12. Since Sue charges \$20 per lawn and uses \$1 per lawn, for each lawn she will earn \$19. Let x represent the number of lawns Sue has cut, and y be the amount she has made in profits: $y = 19x - 665$. The break-even point is when Sue owes nothing and has made nothing, so when $y = 0$.

$$\begin{aligned} y &= 19x - 665 \\ 0 &= 19x - 665 \\ 19x &= 665 \\ x &= 35 \end{aligned}$$

At the break-even point, Sue will have cut 35 lawns.

13. Let x represent the number of chairs made per day and let y represent the number of tables made per day. This solution is worked out using minutes, although hours could also be used. Since both items require 8 min on the lathe then an equation representing the time required would be $8x + 8y = 360$. To write an equation for the sander, multiply each item by the time required: $8x + 20y = 420$.

Isolate x in the first equation: $x = 45 - y$

Substitute and solve:

$$\begin{aligned} 8(45 - y) + 20y &= 420 \\ 360 - 8y + 20y &= 420 \\ 12y &= 60 \\ y &= 5 \end{aligned}$$

Solve for x:

$$\begin{aligned} x &= 45 - y \\ x &= 45 - 5 \\ x &= 40 \end{aligned}$$

They can build 40 chairs and 5 tables.

14. Let x represent the number of grams of soy milk per lunch and let y represent the number of grams of vegetables per lunch. Since both items contain 0.03 g of protein, an equation representing the total amount of protein would be $0.03x + 0.03y = 20$. To write an equation for the amount of carbohydrates, I multiply the number of grams of each item by the portion which is carbohydrates: $0.005x + 0.14y = 50$. Now solve the system of equations.

Isolate x in the first equation: $x = \frac{2000}{3} - y$

Substitute and solve:

$$0.005x + 0.14y = 50$$

$$0.005\left(\frac{2000}{3} - y\right) + 0.14y = 50$$

$$\frac{10}{3} - 0.005y + 0.14y = 50$$

$$0.135y = \frac{140}{3}$$

$$y \doteq 345.679$$

Solve for x:

$$x = \frac{2000}{3} - y$$

$$= \frac{2000}{3} - 345.679$$

$$\doteq 320.988$$

He should eat about 320.988 g of soy milk and about 345.679 g of vegetables.

15. Let E represent Nicole's total earnings and let s represent her sales. The equation representing her earnings at High Tech is $E = 0.05s + 500$ and the equation representing her earnings at Best Computers is $E = 0.075s + 400$. To determine the point where Best Computers becomes the better job, I will solve this system of equations.

$$\begin{aligned} 0.05s + 500 &= 0.075s + 400 \\ 100 &= 0.025s \\ s &= 4000 \end{aligned}$$

Nicole should accept the job at High Tech if she thinks she will make less than \$4000 per week in sales, as she will earn more this way. Otherwise, she should accept the job at Best Computers because she will earn more there.

16. a) In the second step, she incorrectly expands $-(4x - 10)$ as $-4x - 10$; it should be $-4x + 10$. When solving for y, she calculated the value of $4(-7)$, but did not subtract 10.

b) $x = 3$, $y = 2$; I substituted $y = 4x - 10$ into $2x - y = 4$, then solved for x, then substituted $x = 3$ into $y = 4x - 10$ and solved for y.

17. Let x represent the number of nickels and y represent the number of quarters. The number of dimes is $x + y + 5$. The equation to represent the number of coins is $x + y + (5 + x + y) = 49$. The equation to represent the amount she has is $0.05x + 0.25y + 0.10(x + y + 5) = 5.20$.

Solve for x:

$$\begin{aligned} x + y + (5 + x + y) &= 49 \\ 2x + 2y &= 44 \\ 2x &= 44 - 2y \\ x &= 22 - y \end{aligned}$$

Substitute and solve:

$$\begin{aligned} 0.05x + 0.25y + 0.10(5 + x + y) &= 5.20 \\ 0.15x + 0.35y &= 4.70 \\ 0.15(22 - y) + 0.35y &= 4.70 \\ 3.30 + 0.20y &= 4.70 \\ 0.20y &= 1.40 \\ y &= 7 \end{aligned}$$

Solve for x:

$$\begin{aligned} x &= 22 - y \\ x &= 22 - 7 \\ x &= 15 \end{aligned}$$

Now I must determine the number of dimes:

$$\begin{aligned} \text{dimes} &= 5 + x + y \\ &= 5 + 15 + 7 \\ &= 27 \end{aligned}$$

Jennifer has 15 nickels, 27 dimes and 7 quarters.

18. Answers may vary, e.g., I think this strategy is called substitution because it involves two substitutions: the first one to obtain an equation in only one variable, the second one to solve for the other variable.

$$\begin{aligned} x + 4y &= 8 \text{ (equation 1)} \\ 3x - 16y &= 3 \text{ (equation 2)} \\ x &= 8 - 4y \text{ (isolate } x \text{ in equation 1)} \\ 3(8 - 4y) - 16y &= 3 \text{ (first substitution)} \\ y &= 0.75 \text{ (solve for } y) \\ x &= 8 - 4(0.75) \text{ (second substitution)} \\ x &= 5 \text{ (solve for } x) \end{aligned}$$

19. Let x represent the amount Marko invested in bonds and y the amount he deposited in a savings account. The amount he invested in stocks is $4y$. All of his investments combined must add to \$300 000, so $x + y + 4y = 300\,000$. Their combined interest must add to \$35 600, so $0.15(4y) + 0.10x + 0.04y = 35\,600$.
Isolate x: $x = 300\,000 - 5y$
Substitute and solve:

$$\begin{aligned} 0.64y + 0.10x &= 35\,600 \\ 0.64y + 0.10(300\,000 - 5y) &= 35\,600 \\ 0.64y + 30\,000 - 0.5y &= 35\,600 \\ 0.14y &= 5\,600 \\ y &= 40\,000 \end{aligned}$$

Solve for x:

$$\begin{aligned} x &= 300\,000 - 5y \\ x &= 300\,000 - 5(40\,000) \\ x &= 100\,000 \end{aligned}$$

Investment in stocks:

$$\begin{aligned} 4y &= 4(40\,000) \\ 4y &= 160\,000 \end{aligned}$$

Marko invested \$160 000 in stocks, \$100 000 in bonds and \$40 000 in a savings account.

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1. a) The equations are:
(1) $x - 3y = 2$
(2) $2x + y = -5$
Adding equations (1) and (2) gives:
$3x - 2y = -3$
Subtracting equation (2) from equation (1) gives:
$-x - 4y = 7$
b) The equations are:
(1) $x - y = 2$
(2) $2x + 3y = 19$
Adding equations (1) and (2) gives:
$3x + 2y = 21$
Subtracting equation (2) from equation (1) gives:
$-x - 4y = -17$
c) The equations are:
(1) $3x + y = 3$
(2) $x - 2y = 8$
Adding equations (1) and (2) gives:
$4x - y = 11$
Subtracting equation (2) from equation (1) gives:
$2x + 3y = -5$
d) The equations are:
(1) $4x + 2y = 8$
(2) $-x - 2y = -4$
Adding equations (1) and (2) gives:
$3x = 4$
Subtracting equation (2) from equation (1) gives:
$5x + 4y = 12$
2. a) i) The equations are:
(1) $x - 3y = 2$
(2) $2x + y = -5$
Rearrange equation (2) to isolate y.
(3) $y = -2x - 5$
Substitute equation (3) into equation (1) and solve for x.

$$\begin{aligned} x - 3(-2x - 5) &= 2 \\ x + 6x + 15 &= 2 \\ 7x &= -13 \\ x &= -\frac{13}{7} \end{aligned}$$

Substitute this value into equation (3) to solve for y.

$$\begin{aligned} y &= -2\left(-\frac{13}{7}\right) - 5 \\ &= \frac{26}{7} - 5 \\ &= \frac{26}{7} - \frac{35}{7} \\ &= -\frac{9}{7} \end{aligned}$$

The solution to this system is $\left(-\frac{13}{7}, -\frac{9}{7}\right)$.
ii) The equations are:
(1) $x - y = 2$
(2) $2x + 3y = 19$
Rearrange equation (1) to isolate y.
(3) $y = x - 2$
Substitute equation (3) into equation (2) and solve for x.

$$\begin{aligned} 2x + 3(x - 2) &= 19 \\ 2x + 3x - 6 &= 19 \\ 5x &= 25 \\ x &= 5 \end{aligned}$$

Substitute this value into equation (3) to solve for y.

$$\begin{aligned} y &= x - 2 \\ y &= (5) - 2 \\ y &= 3 \end{aligned}$$

The solution to this system is (5, 3).
iii) The equations are:
(1) $3x + y = 3$
(2) $x - 2y = 8$
Rearrange equation (1) to isolate y.
(3) $y = -3x + 3$
Substitute equation (3) into equation (2) and solve for x.

$$\begin{aligned} x - 2(-3x + 3) &= 8 \\ x + 6x - 6 &= 8 \\ 7x &= 14 \\ x &= 2 \end{aligned}$$

Substitute this value into equation (3) to solve for y.

$$\begin{aligned} y &= -3x + 3 \\ y &= -3(2) + 3 \\ y &= -6 + 3 \\ y &= -3 \end{aligned}$$

The solution to this system is (2, –3).
iv) The equations are:
(1) $4x + 2y = 8$
(2) $-x - 2y = -4$
Rearrange equation (1) to isolate y.
(3) $y = -2x + 4$

Substitute equation (3) into equation (2) and solve for x.

$$-x-2y=-4$$
$$-x-2(-2x+4)=-4$$
$$-x+4x-8=-4$$
$$3x=4$$
$$x=\frac{4}{3}$$

Substitute this value into equation (3) to solve for y.

$$y=-2x+4$$
$$y=-2\left(\frac{4}{3}\right)+4$$
$$y=-\frac{8}{3}+4$$
$$y=-\frac{8}{3}+\frac{12}{3}$$
$$y=\frac{4}{3}$$

The solution to this system is $\left(\frac{4}{3}, \frac{4}{3}\right)$.

b) i) Check whether $\left(-\frac{13}{7}, -\frac{9}{7}\right)$, the solution for the original system, satisfies the equations in the new system. The equations are:

(A) $3x - 2y = -3$
(B) $-x - 4y = 7$

Equation (A):

$$LS = 3x-2y$$
$$=3\left(-\frac{13}{7}\right)-2\left(-\frac{9}{7}\right)$$
$$=-\frac{39}{7}+\frac{18}{7}$$
$$=-\frac{21}{7}$$
$$=-3$$
$$RS=-3$$
$$LS=RS$$

Therefore, this point satisfies equation (A).

Equation (B):

$$LS=-x-4y$$
$$=-\left(-\frac{13}{7}\right)-4\left(-\frac{9}{7}\right)$$
$$=\frac{13}{7}+\frac{36}{7}$$
$$=\frac{49}{7}$$
$$=7$$
$$RS=7$$
$$LS=RS$$

Therefore, this point also satisfies equation (B).

$\left(-\frac{13}{7}, -\frac{9}{7}\right)$ is the solution to the new system as well.

ii) Check whether (5, 3), the solution for the original system, satisfies the equations in the new system. The equations are:

(A) $3x + 2y = 21$
(B) $-x - 4y = -17$

Equation (A):

LS $= 3x + 2y$
$= 3(5) + 2(3)$
$= 15 + 6$
$= 21$
RS $= 21$
LS $=$ RS

Therefore, this point satisfies equation (A).

Equation (B):

LS $= -x - 4y$
$= -(5) - 4(3)$
$= -5 - 12$
$= -17$
RS $= -17$
LS $=$ RS

Therefore, this point also satisfies Equation (B). (5, 3) is the solution to the new system as well.

iii) Check whether (2, –3), the solution for the original system, satisfies the equations in the new system. The equations are:

(A) $4x - y = 11$ (B) $2x + 3y = -5$

Equation (A):

LS $= 4x - y$
$= 4(2) - (-3)$
$= 8 + 3$
$= 11$
RS $= 11$
LS $=$ RS

Therefore, this point satisfies Equation (A).

Equation (B):

LS $= 2x + 3y$
$= 2(2) + 3(-3)$
$= 4 - 9$
$= -5$
RS $= -5$
LS $=$ RS

Therefore, this point also satisfies Equation (B). (2, –3) is the solution to the new system as well.

iv) Check whether $\left(\frac{4}{3}, \frac{4}{3}\right)$, the solution for the original system, satisfies the equations in the new system. The equations are:

(A) $3x = 4$
(B) $5x + 4y = 12$

Equation (A):

$$LS=3x$$
$$=3\left(\frac{4}{3}\right)$$
$$=4$$
$$RS=4$$
$$LS=RS$$

Therefore, this point satisfies Equation (A).

Equation (B):

$$LS = 5x + 4y$$
$$= 5\left(\frac{4}{3}\right) + 4\left(\frac{4}{3}\right)$$
$$= \frac{20}{3} + \frac{16}{3}$$
$$= \frac{36}{3}$$
$$= 12$$
$$RS = 12$$
$$LS = RS$$

Therefore, this point also satisfies Equation (B). $\left(\frac{4}{3}, \frac{4}{3}\right)$ is the solution to the new system as well.

3. a) The lines intersect at (4, –1); so, this is the solution to the linear system.

b) The equations are:

(1) $x - 2y = 6$

(2) $2x + 3y = 5$

Adding equations (1) and (2) gives:

(3) $3x + y = 11$

Subtracting equation (2) from equation (1) gives:

(4) $-x - 5y = 1$

c) Equation (3):

$LS = 3x + y$
$= 3(4) - 1$
$= 12 - 1$
$= 11$
$RS = 11$
$LS = RS$

Therefore, this point satisfies equation (3).

Equation (4):

$LS = -x - 5y$
$= -(4) - 5(-1)$
$= -4 + 5$
$= 1$
$RS = 1$
$LS = RS$

Therefore, this point also satisfies equation (4). (4, –1) is the solution to the new system as well. The new system and the original system are equivalent.

4. For example,

a) Multiply the equation for the blue line by 3, producing:

$3x - 6y = 18$

b) *x*-intercept:

Let $y = 0$.

$3x - 6(0) = 18$
$3x = 18$
$x = 6$

y-intercept:

Let $x = 0$.

$3(0) - 6y = 18$
$-6y = 18$
$y = -3$

These intercepts are the same as those on the graph in question 3. It is the same line.

c) Multiply the equation for the red line by 5, producing:

$10x + 15y = 25$

x-intercept: Let $y = 0$.

$10x + 15(0) = 25$
$10x = 25$
$x = \frac{5}{2}$

y-intercept: Let $x = 0$.

$10(0) + 15y = 25$
$15y = 25$
$y = \frac{5}{3}$

These intercepts are the same as those on the graph in question 3. It is the same line.

5. a) The equations are:

(1) $4x + y = 1$

(2) $x - 2y = -11$

Rearrange equation (1) to isolate *y*.

(3) $y = -4x + 1$

Substitute equation (3) into equation (2) and solve for *x*.

$x - 2(-4x + 1) = -11$
$x + 8x - 2 = -11$
$9x = -9$
$x = -1$

Substitute this value into equation (3) to solve for *y*.

$y = -4x + 1$
$y = -4(-1) + 1$
$y = 4 + 1$
$y = 5$

Therefore, the solution to this linear system is (–1, 5).

b) Adding equations (1) and (2) gives:

(4) $5x - y = -10$

Subtracting equation (2) from equation (1) gives:

(5) $3x + 3y = 12$

c) Check whether (–1, 5), the solution for the original system, satisfies the equations in the new system.

Equation (4):

$LS = 5x - y$
$= 5(-1) - (5)$
$= -5 - 5$
$= -10$
$RS = -10$
$LS = RS$

Therefore, this point satisfies equation (4).

Equation (5):

$LS = 3x + 3y$
$= 3(-1) + 3(5)$
$= -3 + 15$
$= 12$
$RS = 12$
$LS = RS$

This point also satisfies equation (5). Thus, (–1, 5) is the solution to the new system as well.

6. a) The original equations are:

$x + 2y = 2$

$-2x - y = 5$

Multiplying the first by 3 gives:

(1) $3x + 6y = 6$

Multiplying the second by 2 gives:

(2) $-4x - 2y = 10$

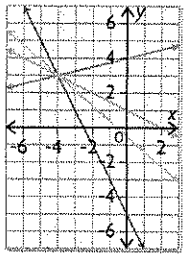
b) Adding equations (1) and (2) gives:

(3) $-x + 4y = 16$

Subtracting equation (2) from equation (1) gives:

(4) $7x + 8y = -4$

c) Graphing equations (1) through (4) produces the following:



All lines intersect at the same point. The two systems are equivalent.

7. a) The equations are:
(1) $-2x + 5y = 2$
(2) $x - 3y = -2$
Rearrange equation (1) to isolate y.

$$(3)\ y = \frac{2}{5}x + \frac{2}{5}$$

Substitute equation (3) into equation (2) and solve for x.

$$x - 3\left(\frac{2}{5}x + \frac{2}{5}\right) = -2$$
$$x - \frac{6}{5}x - \frac{6}{5} = -2$$
$$-\frac{1}{5}x = -\frac{10}{5} + \frac{6}{5}$$
$$-\frac{1}{5}x = -\frac{4}{5}$$
$$x = 4$$

Substitute this value into equation (3) to solve for y.

$$y = \frac{2}{5}x + \frac{2}{5}$$
$$y = \frac{2}{5}(4) + \frac{2}{5}$$
$$y = \frac{8}{5} + \frac{2}{5}$$
$$y = 2$$

The solution to this linear system is (4, 2).

b) Multiplying equation (1) by –3 gives:
(4) $6x - 15y = -6$
Multiply equation (2) by –5 gives:
(5) $-5x + 15y = 10$

c) Adding equations (4) and (5) gives:
(6) $x = 4$
Subtracting equation (5) from equation (4) gives:
(7) $11x - 30y = -16$

d) Check whether (4, 2), the solution for the original system, satisfies the equations in the new system.
Equation (6):

LS = x
= 4
RS = 4
LS = RS

This point satisfies equation (6).

Equation (7):

LS = $11x - 30y$
= 11(4) – 30(2)
= 44 – 60
= –16
RS = –16
LS = RS

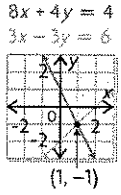
This point also satisfies equation (7). Thus, (4, 2) is the solution to the new system as well. The two systems are equivalent.

8. a) For example, it would not affect the graph because dividing by a non-zero constant is the same as multiplying by its reciprocal.

b) The equations are:
(1) $3x - 3y = 6$
(2) $8x + 4y = 4$
To graph, determine the x-intercepts and y-intercepts.

Equation (1):
y-intercept: Let $x = 0$.
$3(0) - 3y = 6$
$-3y = 6$
$y = -2$
x-intercept: Let $y = 0$.
$3x - 3(0) = 6$
$3x = 6$
$x = 2$

Equation (2):
y-intercept: Let $x = 0$.
$8(0) + 4y = 4$
$4y = 4$
$y = 1$
x-intercept: Let $y = 0$.
$8x + 4(0) = 4$
$8x = 4$
$x = 0.5$

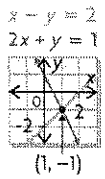


The graphs of the two equations intersect at (1, –1); so, this is the solution to this linear system.

c) Dividing equation (1) by 3 gives:
(3) $x - y = 2$
Dividing equation (2) by 4 gives:
(4) $2x + y = 1$
To graph, determine the x-intercepts and y-intercepts.

Equation (3):
y-intercept: Let $x = 0$.
$(0) - y = 2$
$-y = 2$
$y = -2$
x-intercept: Let $y = 0$.
$x - (0) = 2$
$x = 2$

Equation (4):
y-intercept: Let $x = 0$.
$2(0) + y = 1$
$y = 1$
x-intercept: Let $y = 0$.
$2x + (0) = 1$
$2x = 1$
$x = 0.5$



The graphs of the new equations are exactly the same. The solution to this new system is (1, –1) as well.

d) Adding equations (3) and (4) gives:
(5) $3x = 3$
Subtracting equation (4) from equation (3) gives:
(6) $-x - 2y = 1$

e) Check whether (1, –1), the solution for the original system, satisfies the equations in the new system.
Equation (5):
LS $= 3x$
$= 3(1)$
$= 3$
RS $= 3$
LS = RS
This point satisfies equation (5).
Equation (6):
LS $= -x - 2y$
$= -(1) - 2(-1)$
$= -1 + 2$
$= 1$
RS $= 1$
LS = RS
This point also satisfies equation (6). Thus,
(1, –1) is the solution to the new system as well. The two systems are equivalent.

9. a) The equations are:
(1) $4x + y = 4$
(2) $2x - 3y = -5$
Rearrange equation (1) to isolate y.
(3) $y = -4x + 4$
Substitute equation (3) into equation (2) and solve for x.

$$2x - 3(-4x + 4) = -5$$
$$2x + 12x - 12 = -5$$
$$14x = 7$$
$$x = \frac{1}{2}$$

Substitute this value into equation (3) to solve for y.

$$y = -4x + 4$$
$$y = -4\left(\frac{1}{2}\right) + 4$$
$$y = -2 + 4$$
$$y = 2$$

The solution to this system is $\left(\frac{1}{2}, 2\right)$.

b) Substitute the solution into both sides of this equation to see if it is satisfied.

$$LS = 2x + 11y$$
$$= 2\left(\frac{1}{2}\right) + 11(2)$$
$$= 1 + 22$$
$$= 23$$
$$RS = 23$$
$$LS = RS$$

Therefore, $\left(\frac{1}{2}, 2\right)$ satisfies the equation.

c) If a times equation (1) added to b times equation (2) gives $2x + 11y = 23$, then a times the coefficient of x in equation (1) added to b times the coefficient of x in equation (2) gives 2, the coefficient of x in the resulting equation. Similar statements can be made to relate the coefficients of y and the constants in the three equations. These relationships can be summarized using the following three linear relations.
(4) $4a + 2b = 2$
(5) $a - 3b = 11$
(6) $4a - 5b = 23$
Use substitution on equations (4) and (5) to solve for a and b. Rearrange equation (5) to isolate a.
(7) $a = 3b + 11$
Substitute equation (7) into equation (4) and solve for b.

$$4(3b + 11) + 2b = 2$$
$$12b + 44 + 2b = 2$$
$$14b = -42$$
$$b = -3$$

Substitute this value into equation (7) to solve for a.

$$a = 3b + 11$$
$$a = 3(-3) + 11$$
$$a = -9 + 11$$
$$a = 2$$

It is likely that $a = 2$ and $b = -3$. Check that these values satisfy equation (6) as well, in case there is actually no solution.
LS $= 4a - 5b$
$= 4(2) - 5(-3)$
$= 8 + 15$
$= 23$
RS $= 23$
LS = RS
These values also satisfy equation (6). Thus, $a = 2$ and $b = -3$.

10. For example, **a)** The original equations are:
(1) $3x - 4y = 3$
(2) $-x - y = 6$
Add equations (1) and (2).
(3) $2x - 5y = 9$
Subtract equation (2) from equation (1).
(4) $4x - 3y = -3$
Equations (3) and (4) form a new system.
Multiply equation (1) by 2.
(5) $6x - 8y = 6$
Multiply Equation (2) by –1.
(6) $x + y = -6$
Equations (5) and (6) form another new system.
b) Solve the original system using the method of substitution. Rearrange equation (2) to isolate y.
(7) $y = -x - 6$
Substitute equation (7) into equation (1) and solve for x.

$$3x - 4(-x - 6) = 3$$
$$3x + 4x + 24 = 3$$
$$7x = -21$$
$$x = -3$$

Substitute this value into equation (7) to solve for y.

$$y = -x - 6$$
$$y = -(-3) - 6$$
$$y = 3 - 6$$
$$y = -3$$

So, (–3, –3) is the solution to the original system. Verify that it is the solution to the other systems by confirming that it satisfies their equations.

Equation (3):

LS $= 2x - 5y$
$= 2(-3) - 5(-3)$
$= -6 + 15$
$= 9$
RS $= 9$
LS $=$ RS

This point satisfies equation (3).

Equation (4):

LS $= 4x - 3y$
$= 4(-3) - 3(-3)$
$= -12 + 9$
$= -3$
RS $= -3$
LS $=$ RS

This point also satisfies equation (4).

So, (–3, –3) is the solution to the first new system as well. The two systems are equivalent.

Equation (5):

LS $= 6x - 8y$
$= 6(-3) - 8(-3)$
$= -18 + 24$
$= 6$
RS $= 6$
LS $=$ RS

This point satisfies equation (5).

Equation (6):

LS $= x + y$
$= (-3) + (-3)$
$= -3 - 3$
$= -6$
RS $= -6$
LS $=$ RS

This point also satisfies equation (6). So, (–3, –3) is the solution to the second new system as well. All three systems have the same solution. They are equivalent.

11. For example,

a) Adding the two equations will eliminate the y term from the resulting equivalent equation, allowing you to solve for x. Similarly, subtracting the two equations will eliminate the x term, allowing you to solve for y.

b) Create a pair of equations that have a common term on one coefficient, but of opposite sign. This term will be eliminated when the equations are added. For example,
$4x - 3y = 10$
$-4x - 2y = 3$

c) Create a pair of equations that have a common term on one coefficient. This term will be eliminated when the equations are subtracted. For example,
$4x - 3y = 10$
$7x - 3y = 12$

12. Adding the equations gives:
(1) $4x = 12$
Subtracting the second equation from the first gives:
(2) $6y = -4$
Solve these equations to determine the solution to the system.

$4x = 12$
$x = 3$
$6y = -4$
$y = -\frac{2}{3}$

The solution to this system is $\left(3, -\frac{2}{3}\right)$.

13. a) Let x represent the greater of the numbers, and let y represent the lesser. The question provides the following relationships.
(1) $x + y = 33$
(2) $x - y = 57$

b) Adding equations (1) and (2) gives: (3) $2x = 90$
Subtracting equation (2) from equation (1) gives:
(4) $2y = -24$
Equations (3) and (4) form an equivalent system.

c)
$2x = 90$
$x = 45$
$2y = -24$
$y = -12$

The greater number is 45, and the lesser number is –12.

14. a) Let x represent the number of guests who ordered the chicken dinner, and y represent the number of guests who ordered the fish dinner. The total number of guests who ordered their dinners in advance is 200:
(1) $x + y = 200$
If each chicken dinner costs \$20, then $20x$ represents the bill for chicken dinners. Similarly, $18y$ represents the bill for fish dinners. The total bill is \$3880. Therefore:
(2) $20x + 18y = 3880$
These two linear equations form a system.

b) Multiplying the guest equation equation (1) by 20 gives: (3) $20x + 20y = 4000$
Subtracting the cost equation equation (2) from equation (3) gives:
(4) $2y = 120$
Equations (3) and (4) form an equivalent system.

c) Solve equation (4) to determine the value of y.

$2y = 120$
$y = 60$

Substitute this value into equation (3) to solve for x.

$20x + 20y = 4000$
$20x + 20(60) = 4000$
$20x + 1200 = 4000$
$20x = 2800$
$x = 140$

The guests ordered 140 chicken dinners and 60 fish dinners.

15. a) In system C, $x = -2$ and $y = -4$. These values must satisfy the equations for the other systems in order for these systems to be equivalent. Check whether the equations are satisfied.

System A:

First equation:

LS $= 3x - 2y$
$= 3(-2) - 2(-4)$
$= -6 + 8$
$= 2$
RS $= 2$
LS $=$ RS

The first equation is satisfied.

Second equation:

LS $= -10x + 3y$
$= -10(-2) + 3(-4)$
$= 20 - 12$
$= 8$
RS $= 8$
LS $=$ RS

The second equation is satisfied as well. Systems A and C have the same solution; so, they are equivalent systems.
System B:
First equation:
LS $= -7x + y$
$= -7(-2) + (-4)$
$= 14 - 4$
$= 10$
RS $= 10$
LS $=$ RS
The first equation is satisfied.
Second equation:
LS $= 13x - 5y$
$= 13(-2) - 5(-4)$
$= -26 + 20$
$= -6$
RS $= -6$
LS $=$ RS
The second equation is satisfied as well. Systems A, B, and C all have the same solution; so, they are equivalent systems.
b) For example, multiply the first equations in both systems A and B by 2.
$6x - 4y = 4$
$-14x + 2y = 20$
Systems A and B were equivalent; so, the first equations in each system were also equivalent. Multiplying each of these equations by 2 to generate the equations above produces yet another equivalent system, as required.
16. For example,
a) Equivalent systems of linear equations are ones which have the same solution. The same values will satisfy all of their equations, and their graphs will all intersect at the same point.
b) You can add them, subtract them, or multiply either one by a non-zero constant.
c) This can sometimes help solve the original system by cancelling out one of the variables.
17. a) The equations are:
(1) $x - 4y = -22$
(2) $2x + y = 1$
Multiply equation (2) by 4 to make the coefficients of y equal, but of opposite sign.
(3) $8x + 4y = 4$
Add equations (1) and (3) to eliminate the y term.
(4) $9x = -18$
Similarly, multiply equation (1) by 2 to make the coefficients of x equal.
(5) $2x - 8y = -44$
Subtract equation (2) from equation (5) to eliminate the x term.
(6) $-9y = -45$
Equations (4) and (6) form an equivalent system. Solve these equations to solve the original system.
$9x = -18$
$x = -2$
$-9y = -45$
$y = 5$
The solution to this system is (–2, 5).

b) The equations are:
(1) $3x - 4y = 30$
(2) $2x + 5y = -26$
Multiply equation (1) by 5, and equation (2) by 4 to make the coefficients of y equal, but of opposite sign.
(3) $15x - 20y = 150$
(4) $8x + 20y = -104$
Add equations (3) and (4) to eliminate the y term.
(5) $23x = 46$
Similarly, multiply equation (1) by 2, and equation (2) by 3 to make the coefficients of x equal.
(6) $6x - 8y = 60$
(7) $6x + 15y = -78$
Subtract equation (7) from equation (6) to eliminate the x term.
(8) $-23y = 138$
Equations (5) and (8) form an equivalent system. Solve these equations to solve the original system.
$23x = 46$
$x = 2$
$-23y = 138$
$y = -6$
The solution to this system is (2, –6).
18. a) No. The equations are:
(1) $2x + y = 7$
(2) $8x + 4y = 28$
To eliminate a variable, at least one equation must be multiplied such that the coefficients in front of one variable are either the same or opposites. Attempting to multiply one of these equations to satisfy this condition results in both equations being identical, however. For instance, multiply equation (1) by 4.
(3) $8x + 4y = 28$
The coefficients of both variables are the same. When equation (2) is subtracted from equation (3), both x and y terms are eliminated, and the result is:
(4) $0 = 0$
There are no variables in this equation. Any equivalent system will have equations with either 0 or 2 variables.
b) Equation (4) suggests that this system has a solution whenever $0 = 0$. This is always true; so, the system has infinite solutions.
c) If a linear system has infinite solutions, it means that the lines must intersect at infinitely many points. This is only true if the lines are the same (they are overlapping).
19. No. The equations are:
(1) $2x + y = 7$
(2) $8x + 4y = 10$
To eliminate a variable, at least one equation must be multiplied such that the coefficients in front of one variable are either the same or opposites. Attempting to multiply one of these equations to satisfy this condition results in the coefficients of both variables being identical. For instance, multiply equation (1) by 4.
(3) $8x + 4y = 28$
The coefficients of both variables are the same. When equation (2) is subtracted from equation (3), both the x and y terms are eliminated, and the result is:
(4) $0 = 18$
There are no variables in this equation. Any equivalent system will have equations with either 0 or 2 variables.

b) Equation (4) suggests that this system has a solution whenever $0 = 18$. This is never true, however; so, the system has no solutions.
c) If a system has no solutions, it means that the lines can never intersect. This is only true if the lines are parallel, but not the same (same slope, different y-intercept).

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1. a) Subtract, since the left sides share a $+y$ term.
b) Subtract, since the left sides share a $-2y$ term.
c) Subtract, since the left sides share a $+4x$ term.
d) Add, since the left sides share $5y$ terms of opposite sign.
2. a) Multiply equation (1) by 3 and equation (2) by 4 to create common $+12x$ terms on the left sides. Subtract one equation from the other to eliminate these terms.
b) Subtract one equation from the other, since the left sides share a $+4x$ term.
3. Let x represent the hourly rate for the welder, and y represent the hourly rate for the apprentice. Then, the following linear relations express what is known about their earnings.
(1) $3x + 5y = 175$
(2) $7x + 8y = 346$
Multiply equation (1) by 8 and equation (2) by 5.
(3) $24x + 40y = 1400$
(4) $35x + 40y = 1730$
Subtract equation (3) from equation (4) and solve for x.

$$\begin{aligned} 11x &= 330 \\ x &= 30 \end{aligned}$$

Substitute this value into equation (1) to solve for y.

$$\begin{aligned} 3x + 5y &= 175 \\ 3(30) + 5y &= 175 \\ 90 + 5y &= 175 \\ 5y &= 85 \\ y &= 17 \end{aligned}$$

The welder earns \$30/h, and the apprentice earns \$17/h.
4. a) Multiply equation (1) by 2 and equation (2) by 1, such that the left sides share $4y$ terms of opposite sign.
b) Multiply equation (1) by 8 and equation (2) by 7, such that the left sides share $56y$ terms of opposite sign.
c) Multiply equation (1) by 5 and equation (2) by 3, such that the left sides share $+15y$ terms.
d) Multiply equation (1) by 1 and equation (2) by 2, such that the left sides share $4y$ terms of opposite sign.
5. a) Multiply equation (1) by 3 and equation (2) by 4, such that the left sides share $+12x$ terms.
b) Multiply equation (1) by 5 and equation (2) by 3, such that the left sides share $+15x$ terms.
c) Multiply equation (1) by 1 and equation (2) by 2, such that the left sides share $4x$ terms of opposite sign.
d) Multiply equation (1) by 1 and equation (2) by 3, such that the left sides share $+9x$ terms.
6. a) The equations are:
(1) $3x + y = -2$
(2) $x - y = -6$
Add equations (1) and (2) and solve for x.

$$\begin{aligned} 4x &= -8 \\ x &= -2 \end{aligned}$$

Substitute this value into equation (2) to solve for y.

$$\begin{aligned} x - y &= -6 \\ -2 - y &= -6 \\ y &= -2 + 6 \\ y &= 4 \end{aligned}$$

The solution to this system is $(-2, 4)$.
b) The equations are:
(1) $x + 5y = 1$
(2) $2x + 3y = 9$
Multiply equation (1) by 2.
(3) $2x + 10y = 2$
Subtract equation (2) from equation (3).

$$\begin{aligned} 7y &= -7 \\ y &= -1 \end{aligned}$$

Substitute this value into equation (1) to solve for x.

$$\begin{aligned} x + 5(-1) &= 1 \\ x - 5 &= 1 \\ x &= 6 \end{aligned}$$

The solution to this system is $(6, -1)$.
c) The equations are:
(1) $4x - y = 5$
(2) $-5x + 2y = -1$
Multiply equation (1) by 2.
(3) $8x - 2y = 10$
Add equations (2) and (3) and solve for x.

$$\begin{aligned} 3x &= 9 \\ x &= 3 \end{aligned}$$

Substitute this value into equation (1) to solve for y.

$$\begin{aligned} 4x - y &= 5 \\ 4(3) - y &= 5 \\ 12 - y &= 5 \\ y &= 12 - 5 \\ y &= 7 \end{aligned}$$

The solution to this system is $(3, 7)$.
d) The equations are:
(1) $2x - 3y = -2$
(2) $3x - y = 0.5$
Multiply equation (2) by 3.
(3) $9x - 3y = 1.5$
Subtract equation (1) from equation (3) and solve for x.

$$\begin{aligned} 7x &= 3.5 \\ x &= 0.5 \end{aligned}$$

Substitute this value into equation (2) to solve for y.

$$\begin{aligned} 3x - y &= 0.5 \\ 3(0.5) - y &= 0.5 \\ 1.5 - y &= 0.5 \\ y &= 1.5 - 0.5 \\ y &= 1 \end{aligned}$$

The solution to this system is $(0.5, 1)$.
e) The equations are:
(1) $3x - 2y = -39$
(2) $x + 3y = 31$
Multiply equation (2) by 3.
(3) $3x + 9y = 93$
Subtract equation (1) from equation (3) and solve for y.

$$\begin{aligned} 11y &= 132 \\ y &= 12 \end{aligned}$$

Substitute this value into equation (2) to solve for x.

$$\begin{aligned} x + 3y &= 31 \\ x + 3(12) &= 31 \\ x + 36 &= 31 \\ x &= -5 \end{aligned}$$

The solution to this system is (–5, 12).

f) The equations are:

(1) $5x - y = -3.8$

(2) $4x + 3y = 7.6$

Multiply equation (1) by 3.

(3) $15x - 3y = -11.4$

Add equations (2) and (3).

$$\begin{aligned} 19x &= -3.8 \\ x &= -0.2 \end{aligned}$$

Substitute this value into equation (1) to solve for y.

$$\begin{aligned} 5x - y &= -3.8 \\ 5(-0.2) - y &= -3.8 \\ -1 - y &= -3.8 \\ y &= 3.8 - 1 \\ y &= 2.8 \end{aligned}$$

The solution to this system is (–0.2, 2.8).

7. The equations are:

(1) $x + 3y = -1$

(2) $4x - y = 22$

Multiply equation (2) by 3.

(3) $12x - 3y = 66$

Add equations (1) and (3) and solve for x.

$$\begin{aligned} 13x &= 65 \\ x &= 5 \end{aligned}$$

Substitute this value into equation (2) to solve for y.

$$\begin{aligned} 4x - y &= 22 \\ 4(5) - y &= 22 \\ 20 - y &= 22 \\ y &= 20 - 22 \\ y &= -2 \end{aligned}$$

So, (5, –2) is the point of intersection for these two lines.

To verify, check that this point satisfies both equations.

Equation (1):

$$\begin{aligned} \text{LS} &= x + 3y \\ &= (5) + 3(-2) \\ &= 5 - 6 \\ &= -1 \\ \text{RS} &= -1 \\ \text{LS} &= \text{RS} \end{aligned}$$

This point satisfies equation (1).

Equation (2):

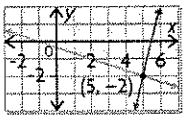
$$\begin{aligned} \text{LS} &= 4x - y \\ &= 4(5) - (-2) \\ &= 20 + 2 \\ &= 22 \\ \text{RS} &= 22 \\ \text{LS} &= \text{RS} \end{aligned}$$

This point also satisfies equation (2). So,

(5, –2) is the point of intersection, since it satisfies both equations, meaning it is common to both lines. This assertion can also be verified by graphing.

$y = 4x - 22$

$y = -\frac{1}{3} - \frac{1}{3}x$



This graph shows that the point of intersection is (5, –2).

8. a) Let x represent the distance walked by Lori, and y represent the distance walked by Nicholas. They walked a total of 72.7 km, so:

(1) $x + y = 72.7$

Also, Lori walked 8.9 km farther than Nicholas, so:

(2) $x - y = 8.9$

b) Add equations (1) and (2) and solve for x.

$$\begin{aligned} 2x &= 81.6 \\ x &= 40.8 \end{aligned}$$

Substitute this value into equation (2) to solve for y.

$$\begin{aligned} x - y &= 8.9 \\ (40.8) - y &= 8.9 \\ y &= 40.8 - 8.9 \\ y &= 31.9 \end{aligned}$$

Lori walked 40.8 km, and Nicholas walked 31.9 km.

9. a) Let l represent the length, and let w represent the width. The total perimeter is 54 m, so:

(1) $2l + 2w = 54$

Also, the difference between the length and width is 9 m. The length is longer, so (2) $l - w = 9$

b) Multiply equation (2) by 2.

(3) $2l - 2w = 18$

Add equations (1) and (3) and solve for l.

$$\begin{aligned} 4l &= 72 \\ l &= 18 \end{aligned}$$

Substitute this value into equation (2) to solve for w.

$$\begin{aligned} l - w &= 9 \\ (18) - w &= 9 \\ w &= 18 - 9 \\ w &= 9 \end{aligned}$$

The court has a length of 18 m and a width of 9 m.

10. Let x represent the grams of chocolate that is 99% cocoa, and y represent the grams of chocolate that is 70% cocoa. Rolf needs a total of 500 g of chocolate, and so: (1) $x + y = 500$

If the chocolate required for the recipe is 86% cocoa, then it has $0.86 \times 500 = 430$ g of cocoa total. The mass of cocoa from the 99% cocoa chocolate is $0.99x$, and the mass of cocoa from the 70% cocoa chocolate is $0.70y$. So:

(2) $0.99x + 0.70y = 430$

Multiply equation (1) by 0.99.

(3) $0.99x + 0.99y = 495$

Subtract equation (2) from equation (3) and solve for y.

$$\begin{aligned} 0.29y &= 65 \\ y &\doteq 224 \end{aligned}$$

Put this value into equation (1) to solve for x.

$$\begin{aligned} x + y &= 500 \\ x + (224) &\doteq 500 \\ x &\doteq 276 \end{aligned}$$

Rolf needs about 276 g of chocolate that is 99% cocoa, and about 224 g of chocolate that is 70% cocoa for his 86% cocoa blend.

11. a) The equations are:
(1) $4x + 7y = 23$
(2) $6x - 5y = -12$
Multiply equation (1) by 5, and multiply equation (2) by 7.
(3) $20x + 35y = 115$
(4) $42x - 35y = -84$
Add equations (3) and (4) and solve for x.

$$\begin{aligned} 62x &= 31 \\ x &= 0.5 \end{aligned}$$

Substitute this value into equation (1) to solve for y.

$$\begin{aligned} 4x + 7y &= 23 \\ 4(0.5) + 7y &= 23 \\ 2 + 7y &= 23 \\ 7y &= 21 \\ y &= 3 \end{aligned}$$

The lines intersect at (0.5, 3). Verify this by checking whether this point satisfies the equations for each line.
Equation (1):

$$\begin{aligned} \text{LS} &= 4x + 7y \\ &= 4(0.5) + 7(3) \\ &= 2 + 21 \\ &= 23 \\ \text{RS} &= 23 \\ \text{LS} &= \text{RS} \end{aligned}$$

This point satisfies equation (1).
Equation (2):

$$\begin{aligned} \text{LS} &= 6x - 5y \\ &= 6(0.5) - 5(3) \\ &= 3 - 15 \\ &= -12 \\ \text{RS} &= -12 \\ \text{LS} &= \text{RS} \end{aligned}$$

The point also satisfies equation (2). So, (0.5, 3) is the point of intersection since it satisfies both equations.
b) The equations are:

$$(1)\ \frac{x}{11} - \frac{y}{8} = -2$$

$$(2)\ \frac{x}{2} - \frac{y}{4} = 3$$

Multiply equation (1) by 88, and multiply equation (2) by 16.
(3) $8x - 11y = -176$
(4) $8x - 4y = 48$
Subtract equation (3) from equation (4) and solve for y.

$$\begin{aligned} 7y &= 224 \\ y &= 32 \end{aligned}$$

Substitute this value into equation (4) to solve for x.

$$\begin{aligned} 8x - 4y &= 48 \\ 8x - 4(32) &= 48 \\ 8x - 128 &= 48 \\ 8x &= 176 \\ x &= 22 \end{aligned}$$

The lines intersect at (22, 32). Verify this by checking whether this point satisfies the equation for each line.
Equation (1):

$$\begin{aligned} \text{LS} &= \frac{x}{11} - \frac{y}{8} \\ &= \frac{(22)}{11} - \frac{(32)}{8} \\ &= 2 - 4 \\ &= -2 \\ \text{RS} &= -2 \\ \text{LS} &= \text{RS} \end{aligned}$$

This point satisfies equation (1).
Equation (2):

$$\begin{aligned} \text{LS} &= \frac{x}{2} - \frac{y}{4} \\ &= \frac{(22)}{2} - \frac{(32)}{4} \\ &= 11 - 8 \\ &= 3 \\ \text{RS} &= 3 \\ \text{LS} &= \text{RS} \end{aligned}$$

The point also satisfies equation (2). Since it satisfies both equations, (22, 32) is the point of intersection.
c) The equations are:
(1) $0.5x - 0.3y = 1.5$
(2) $0.2x - 0.1y = 0.7$
Multiply equation (2) by 3.
(3) $0.6x - 0.3y = 2.1$
Subtract equation (1) from equation (3) and solve for x.

$$\begin{aligned} 0.1x &= 0.6 \\ x &= 6 \end{aligned}$$

Substitute this value into equation (2) to solve for y.

$$\begin{aligned} 0.2x - 0.1y &= 0.7 \\ 0.2(6) - 0.1y &= 0.7 \\ 1.2 - 0.1y &= 0.7 \\ 0.1y &= 1.2 - 0.7 \\ 0.1y &= 0.5 \\ y &= 5 \end{aligned}$$

The lines intersect at (6, 5). Verify this by checking whether this point satisfies the equation for each line.
Equation (1):

$$\begin{aligned} \text{LS} &= 0.5x - 0.3y \\ &= 0.5(6) - 0.3(5) \\ &= 3 - 1.5 \\ &= 1.5 \\ \text{RS} &= 1.5 \\ \text{LS} &= \text{RS} \end{aligned}$$

This point satisfies equation (1).
Equation (2):

$$\begin{aligned} \text{LS} &= 0.2x - 0.1y \\ &= 0.2(6) - 0.1(5) \\ &= 1.2 - 0.5 \\ &= 0.7 \\ \text{RS} &= 0.7 \\ \text{LS} &= \text{RS} \end{aligned}$$

The point also satisfies equation (2). Since it satisfies both equations, (6, 5) is the point of intersection.

d) The equations are:

(1) $\frac{x}{2} - 5y = 7$

(2) $3x + \frac{y}{2} = \frac{23}{2}$

Multiply equation (1) by 2, and multiply equation (2) by 20.

(3) $x - 10y = 14$

(4) $60x + 10y = 230$

Add equations (3) and (4) and solve for x.

$$\begin{aligned} 61x &= 244 \\ x &= 4 \end{aligned}$$

Substitute this value into equation (3) to solve for y.

$$\begin{aligned} x - 10y &= 14 \\ (4) - 10y &= 14 \\ 10y &= 4 - 14 \\ 10y &= -10 \\ y &= -1 \end{aligned}$$

The lines intersect at $(4, -1)$. Verify this by checking whether this point satisfies the equation for each line.

Equation (1):

$$\begin{aligned} \text{LS} &= \frac{x}{2} - 5y \\ &= \frac{(4)}{2} - 5(-1) \\ &= 2 + 5 \\ &= 7 \end{aligned}$$

$\text{RS} = 7$

$\text{LS} = \text{RS}$

This point satisfies equation (1).

Equation (2):

$$\begin{aligned} \text{LS} &= 3x + \frac{y}{2} \\ &= 3(4) + \frac{(-1)}{2} \\ &= 12 - \frac{1}{2} \\ &= \frac{24}{2} - \frac{1}{2} \\ &= \frac{23}{2} \end{aligned}$$

$\text{RS} = \frac{23}{2}$

$\text{LS} = \text{RS}$

So, the point also satisfies equation (2). Since it satisfies both equations, $(4, -1)$ is the point of intersection.

e) The equations are:

(1) $5x - 12y = 1$

(2) $13x + 9y = 16$

Multiply equation (1) by 3, and multiply equation (2) by 4.

(3) $15x - 36y = 3$

(4) $52x + 36y = 64$

Add equations (3) and (4) and solve for x.

$$\begin{aligned} 67x &= 67 \\ x &= 1 \end{aligned}$$

Substitute this value into equation (1) to solve for y.

$$\begin{aligned} 5x - 12y &= 1 \\ 5(1) - 12y &= 1 \\ -12y &= -4 \\ y &= \frac{1}{3} \end{aligned}$$

The lines intersect at $\left(1, \frac{1}{3}\right)$. Verify this by checking whether this point satisfies the equation for each line.

Equation (1):

$$\begin{aligned} \text{LS} &= 5x - 12y \\ &= 5(1) - 12\left(\frac{1}{3}\right) \\ &= 5 - 4 \\ &= 1 \end{aligned}$$

$\text{RS} = 1$

$\text{LS} = \text{RS}$

This point satisfies equation (1).

Equation (2):

$$\begin{aligned} \text{LS} &= 13x + 9y \\ &= 13(1) + 9\left(\frac{1}{3}\right) \\ &= 13 + 3 \\ &= 16 \end{aligned}$$

$\text{RS} = 16$

$\text{LS} = \text{RS}$

The point also satisfies equation (2). Since it satisfies both equations, $\left(1, \frac{1}{3}\right)$ is the point of intersection.

f) The equations are:

(1) $\frac{x}{9} + \frac{y-3}{3} = 1$

(2) $\frac{x}{2} - (y + 9) = 0$

Multiply equation (1) by 9, and multiply equation (2) by 2.

(3) $x + 3(y - 3) = 9$

(4) $x - 2(y + 9) = 0$

Subtract equation (4) from equation (3) and solve for y.

$$\begin{aligned} 3(y - 3) + 2(y + 9) &= 9 \\ 3y - 9 + 2y + 18 &= 9 \\ 5y &= 0 \\ y &= 0 \end{aligned}$$

Substitute this value into equation (3) to solve for x.

$$\begin{aligned} x + 3(y - 3) &= 9 \\ x + 3[(0) - 3] &= 9 \\ x - 9 &= 9 \\ x &= 18 \end{aligned}$$

The lines intersect at $(18, 0)$. Verify this by checking whether this point satisfies the equation for each line.

Equation (1):

$$LS = \frac{x}{9} + \frac{y-3}{3}$$
$$= \frac{18}{9} + \frac{0-3}{3}$$
$$= 2 - 1$$
$$= 1$$
$$RS = 1$$
$$LS = RS$$

This point satisfies equation (1).
Equation (2):

$$LS = \frac{x}{2} - (y+9)$$
$$= \frac{18}{2} - (0+9)$$
$$= 9 - 9$$
$$= 0$$
$$RS = 0$$
$$LS = RS$$

The point also satisfies equation (2). Since it satisfies both equations, (18, 0) is the point of intersection.

12. Let x represent the grams of mandarin oranges, and y represent the grams of tomatoes. Then, $0.26x$ represents the milligrams of vitamin C from oranges, and $0.13y$ represents the milligrams of vitamin C from tomatoes. Also, $0.13x$ represents the milligrams of vitamin A from oranges, and $0.42y$ represents the milligrams of vitamin A from tomatoes. If there are to be 13 mg of vitamin C and 20.7 mg of vitamin A, then:
(1) $0.26x + 0.13y = 13$
(2) $0.13x + 0.42y = 20.7$
Multiply equation (2) by 2.
(3) $0.26x + 0.84y = 41.4$
Subtract equation (1) from equation (3) and solve for y.

$$0.71y = 28.4$$
$$y = 40$$

Substitute this value into equation (1) to solve for x.

$$0.26x + 0.13y = 13$$
$$0.26x + 0.13(40) = 13$$
$$0.26x + 5.2 = 13$$
$$0.26x = 7.8$$
$$x = 30$$

30 g of mandarin oranges and 40 g of tomatoes have the necessary amounts of vitamin A and vitamin C.

13. Let x represent the amount of time Carl spent running on trails, and y represent the amount of time Carl spent running on rough terrain. Then $10x$ represents the distance Carl ran on trails, since he runs at 10 km/h on trails. Also, $5y$ represents the distance Carl ran on rough terrain, since he runs at 5 km/h on rough terrain. If he ran 12 km in 1.5 h, then:
(1) $x + y = 1.5$ (2) $10x + 5y = 12$
Multiply equation (1) by 10.
(3) $10x + 10y = 15$
Subtract equation (2) from equation (3) and solve for y.
$5y = 3$

$$y = \frac{3}{5}$$

Carl spent $\frac{3}{5}$ of an hour running on rough terrain. The question asked how far he ran,so multiply the time by the speed he was going: 5km/h.

$$5y = 5\left(\frac{3}{5}\right)$$
$$= 3$$

Carl ran 3 km on rough terrain.

14. Let x represent one numerator, and let y represent the other. The question gives two equations.

(1) $\frac{x}{3} + \frac{y}{4} = \frac{17}{12}$

(2) $\frac{y}{3} + \frac{x}{4} = \frac{3}{2}$

To eliminate the fractions and the y terms, multiply equation (1) by 48 and equation (2) by 36.
(3) $16x + 12y = 68$
(4) $12y + 9x = 54$
Subtract equation (4) from equation (3).

$$7x = 14$$
$$x = 2$$

Substitute this value into equation (4) to solve for y.

$$12y + 9x = 54$$
$$12y + 9(2) = 54$$
$$12y + 18 = 54$$
$$12y = 36$$
$$y = 3$$

Therefore $x = 2$, and $y = 3$. Then, the two original fractions are $\frac{2}{3}$ and $\frac{3}{4}$, since these are the two that must sum to $\frac{17}{12}$.

15. Let x represent the amount of money that the students placed in a savings account, and let y represent the amount of money the students placed in a government bond. They invested a total of $6500, so:
(1) $x + y = 6500$
If the savings account has a 3% interest rate, then the interest earned from the money placed there is $0.03x$ after one year. Similarly, the money placed in the government bond accumulates $0.04y$ of interest. The total earned was $235, so:
(2) $0.03x + 0.04y = 235$
Multiply equation (1) by 0.04.
(3) $0.04x + 0.04y = 260$
Subtract equation (2) from equation (3) and solve for x.

$$0.01x = 25$$
$$x = 2500$$

Substitute this value into equation (1) to solve for y.

$$x + y = 6500$$
$$(2500) + y = 6500$$
$$y = 4000$$

The students invested $2500 at the 3% interest rate and $4000 at the 4% interest rate.

16. a) Let x represent the number of students who ordered chicken, and let y represent the number of students who ordered pasta. There are 240 students in total, so: (1) $x + y = 240$
Chicken dinners cost \$12 each, so if x students ordered chicken dinners, the cost is $12x$. Similarly, the cost for pasta dinners is $8y$. The total cost is \$2100, so:
(2) $12x + 8y = 2100$
Multiply equation (1) by 8.
(3) $8x + 8y = 1920$
Subtract equation (3) from equation (2) and solve for x.

$$\begin{aligned} 4x &= 180 \\ x &= 45 \end{aligned}$$

Therefore, 45 students ordered chicken dinners.
b) Substitute the value of x into equation (1) to solve for y.

$$\begin{aligned} x + y &= 240 \\ (45) + y &= 240 \\ y &= 195 \end{aligned}$$

Therefore, 195 students ordered pasta dinners.
17. a) Rows, columns, and diagonals must sum to the same amount. So, from the first row and first column of the first square: $16 + 2 + B = 16 + A + 8$. Simplify to:
(1) $B - A = 6$
From the first row and first column of the second square:

$$24 + \frac{A}{2} + 18 = 24 + 9 + B$$

$$(2)\ \frac{A}{2} - B = -9$$

b) Add equations (1) and (2) and solve for A.

$$\frac{A}{2} - A = -3$$

$$-\frac{A}{2} = -3$$

$$A = 6$$

Substitute this value into equation (1) to solve for B.

$$\begin{aligned} B - A &= 6 \\ B - (6) &= 6 \\ B &= 12 \end{aligned}$$

Therefore, A = 6 and B = 12.
18. Answers may vary, e.g., eliminating a variable means creating an equation with the same solution which has one less variable than the original system. To eliminate a variable from a system, choose one variable, and determine the lowest common multiple between the coefficients on that variable in each equation, without worrying about the sign. Then, multiply each equation by a constant to produce that multiple. Using the given equations:
(1) $3x + 7y = 31$ (2) $5x - 8y = 91$
The coefficients of x have a lowest common multiple of 15. Multiplying equation (1) by 5, and multiplying equation (2) by 3 gives:
(3) $15x + 35y = 155$ (4) $15x - 24y = 273$
Subtracting one equation from the other will eliminate the x variable in the resulting equation. For instance, subtracting equation (4) from equation (3) produces:
$59y = -118$
The coefficients of y in the original equations have a lowest common multiple of 56, ignoring signs. Multiplying equation (1) by 8, and multiplying equation (2) by 7 gives:
(5) $24x + 56y = 248$ (6) $35x - 56y = 637$
Adding equations (5) and (6) will eliminate the y variable in the resulting equation, producing $59x = 885$.
19. Let x represent one of the numbers, and y represent the other. Then the question gives the equations:
(1) $x^2 + y^2 = 74$
(2) $x^2 - y^2 = 24$
Add equations (1) and (2) to eliminate the y^2 variable, and solve for x.

$$\begin{aligned} 2x^2 &= 98 \\ x^2 &= 49 \end{aligned}$$

x is restricted to being a negative number in the question; so, take only the negative root.

$$x = -7$$

Substitute this value into equation (1) to solve for y.

$$\begin{aligned} x^2 + y^2 &= 74 \\ (-7)^2 + y^2 &= 74 \\ 49 + y^2 &= 74 \\ y^2 &= 25 \end{aligned}$$

y is also restricted to being a negative number in the question; so, again, take only the negative root.

$$y = -5$$

The numbers are –5 and –7.
20. The equations are:
(1) $2xy + 3 = 4y$
(2) $3xy + 2 = 5y$
Multiply equation (1) by 3, and multiply equation (2) by 2, to create common coefficients for the xy terms.
(3) $6xy + 9 = 12y$
(4) $6xy + 4 = 10y$
Subtract equation (4) from equation (3) to eliminate the xy terms, and solve for y.

$$5 = 2y$$

$$y = \frac{5}{2}$$

Substitute this value into equation (1) to solve for x.

$$2xy + 3 = 4y$$

$$2x\left(\frac{5}{2}\right) + 3 = 4\left(\frac{5}{2}\right)$$

$$5x + 3 = 10$$

$$5x = 7$$

$$x = \frac{7}{5}$$

The solution to this system is $x = \frac{7}{5}$ and $y = \frac{5}{2}$.

21. a) The equations are:
(1) $ax + by = e$
(2) $cx + dy = f$
Multiply equation (1) by c, and multiply equation (2) by a, in order to create common coefficients for the x terms.
(3) $cax + cby = ce$
(4) $cax + day = fa$

Subtract equation (4) from equation (3) to eliminate the x variables, and solve for y.

$$cby - day = ce - fa$$

$$(bc - ad)y = ce - af$$

$$y = \frac{ce - af}{bc - ad}$$

Substitute this value into equation (1) to solve for x.

$$ax + by = e$$

$$ax + b\left(\frac{ce - af}{bc - ad}\right) = e$$

$$ax = e - \frac{b(ce - af)}{bc - ad}$$

$$ax = \frac{e(bc - ad)}{bc - ad} - \frac{b(ce - af)}{bc - ad}$$

$$ax = \frac{(bce - ade) - (bce - abf)}{bc - ad}$$

$$ax = \frac{bce - bce - ade + abf}{bc - ad}$$

$$ax = \frac{-ade + abf}{bc - ad}$$

$$ax = \frac{-a(de - bf)}{bc - ad}$$

$$x = \frac{de - bf}{-(bc - ad)}$$

$$x = \frac{de - bf}{ad - bc}$$

So, for two linear equations of the general form given, $x = \frac{de - bf}{ad - bc}$ and $y = \frac{ce - af}{bc - ad}$.

b) It is impossible to divide by zero, so solutions for both x and y do not exist when $ad = bc$.

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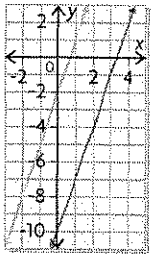
1. Answers may vary, e.g.,

a)

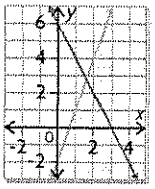
$y = 3x - 2$
$y = 3x - 10$



This linear system has no solutions, since the lines are parallel and never cross.

b)

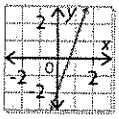
$y = 3x - 2$
$y = -2x + 6$



This linear system has one solution, since the lines intersect at one point.

c)

$y = 3x - 2$
$3y = 9x - 6$



This linear system has infinitely many solutions, since the lines lie on top of one another.

2. For example,

a) i) $3x + 4y = -3$

There are no solutions to this system, because the two equations have equal coeffecients but different constants.

ii) $5x + y = 9$

There is exactly one solution to this system because the coefficients in this equation are not equivalent to equal multiples of the coefficients in the original equation.

iii) $6x + 8y = 4$

There are infinitely many solutions to this system because the coefficients and constants in this equation are exactly twice those in the original equation.

b) i) The equations are:

(1) $3x + 4y = 2$

(2) $3x + 4y = -3$

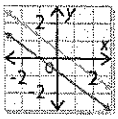
Subtract equation (2) from equation (1).

$0 = 5$

These linear relations have a solution whenever $0 = 5$. Since this is never true, the system has no solutions.

Graphing these equations gives:

$3x + 4y = 2$
$3x + 4y = -3$



The lines never intersect; so, this system has no solutions.

ii) The equations are:

(1) $3x + 4y = 2$ (2) $5x + y = 9$

Multiply equation (2) by 4.

(3) $20x + 4y = 36$

Subtract equation (1) from equation (3) and solve for x.

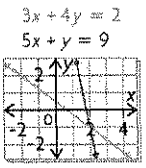
$$17x = 34$$

$$x = 2$$

Substitute this value into equation (2) to solve for y.

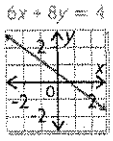
$$\begin{aligned} 5x + y &= 9 \\ 5(2) + y &= 9 \\ 10 + y &= 9 \\ y &= -1 \end{aligned}$$

This system has exactly one solution, $(2, -1)$.
Graphing these equations gives:



The lines intersect at $(2, -1)$, which confirms that this is the one solution.

iii) The equations are:
(1) $3x + 4y = 2$
(2) $6x + 8y = 4$
Multiply equation (1) by 2.
(3) $6x + 8y = 4$
Subtract equation (3) from equation (2).
$0 = 0$
These linear relations will have the same solution whenever $0 = 0$. Since 0 always equals 0, this system has infinitely many solutions.
Graphing these equations gives:



The lines are identical and lie on top of one another, so they intersect at infinitely many points.

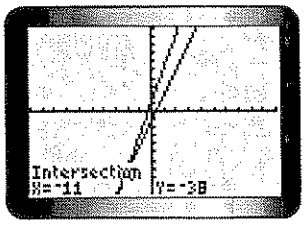
3. a) One solution. The equations are:
(1) $y = 3x - 5$
(2) $y = 4x + 6$
Substitute equation (1) into equation (2) and solve for x.

$$\begin{aligned} 3x - 5 &= 4x + 6 \\ x &= -11 \end{aligned}$$

Substitute this value into equation (1) to solve for y.

$$\begin{aligned} y &= 3x - 5 \\ y &= 3(-11) - 5 \\ y &= -33 - 5 \\ y &= -38 \end{aligned}$$

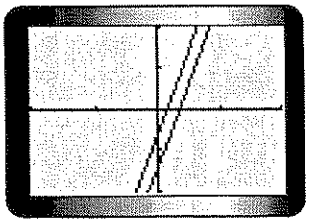
This system has exactly one solution, $(-11, -38)$.
Graphing the system on a graphing calculator confirms this result.



b) No solutions. The equations are:
(1) $y = 4x - 3$
(2) $y = 4x - 7$
Substitute equation (1) into equation (2) and solve for x.

$$\begin{aligned} 4x - 3 &= 4x - 7 \\ -3 &= -7 \end{aligned}$$

This system has a solution whenever $-3 = -7$. This is never true, however, and so this system has no solutions. Graphing the system on a graphing calculator confirms this result.



c) Infinitely many solutions. The equations are:

(1) $y = 5x - \frac{3}{2}$

(2) $y = 5x - 1.5$

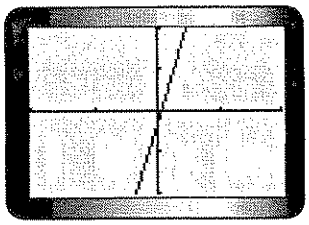
Subtract equation (2) from equation (1).

$$0 = -\frac{3}{2} - (-1.5)$$

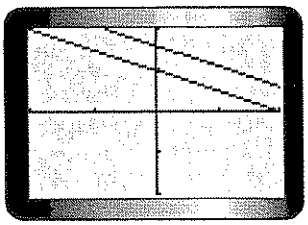
$$0 = -1.5 + 1.5$$

$$0 = 0$$

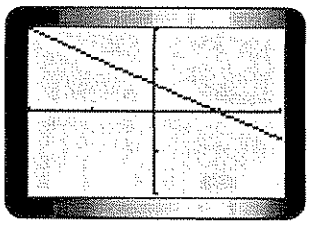
This system has a solution whenever $0 = 0$. This is always true, however, and so this system has infinitely many solutions. Graphing the system on a graphing calculator confirms this result.



d) No solutions. The equations are:
(1) $x + 2y = 10$
(2) $y = 8 - 0.5x$
Rearrange equation (2).
(3) $0.5x + y = 8$
Multiply equation (3) by 2.
(4) $x + 2y = 16$
Subtract equation (1) from equation (4).
$0 = 6$
This system has a solution whenever $0 = 6$. This is never true, however, and so this system has no solutions.
Graphing the system on a graphing calculator confirms this result.



e) Infinitely many solutions. The equations are:
(1) $2x + 3y = 10$
(2) $10x + 15y = 50$
Multiply equation (1) by 5.
(3) $10x + 15y = 50$
Subtract equation (2) from equation (3): $0 = 0$
This system has a solution whenever $0 = 0$. This is always true, however, and so this system has infinitely many solutions. Graphing the system on a graphing calculator confirms this result.



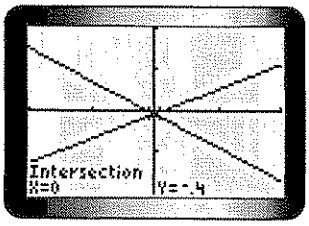
f) One solution. The equations are:
(1) $3x - 5y - 2 = 0$
(2) $4x + 5y + 2 = 0$
Add equations (1) and (2) and solve for x.

$$x = 0$$

Substitute this value into equation (1) to solve for y.

$$3x - 5y - 2 = 0$$
$$3(0) - 5y - 2 = 0$$
$$-5y - 2 = 0$$
$$-5y = 2$$
$$y = -\frac{2}{5}$$

This system has exactly one solution, $\left(0, -\frac{2}{5}\right)$ or $(0, -0.4)$. Graphing the system on a graphing calculator confirms this result.



g) One solution. The equations are:
(1) $y = 1.25x - 0.375$
(2) $5y = 4x$
Convert equation (1) such that decimals are expressed as fractions.

(3) $y = \frac{5}{4}x - \frac{3}{8}$

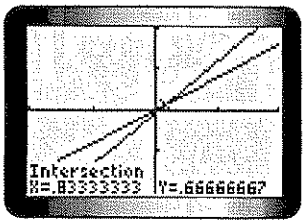
Substitute equation (3) into equation (2) and solve for x.

$$5\left(\frac{5}{4}x - \frac{3}{8}\right) = 4x$$
$$\frac{25}{4}x - \frac{15}{8} = \frac{16}{4}x$$
$$\frac{25}{4}x - \frac{16}{4}x = \frac{15}{8}$$
$$\frac{9}{4}x = \frac{15}{8}$$
$$x = \frac{5}{6}$$

Substitute this value into equation (3) to solve for y.

$$y = \frac{5}{4}x - \frac{3}{8}$$
$$y = \frac{5}{4}\left(\frac{5}{6}\right) - \frac{3}{8}$$
$$y = \frac{25}{24} - \frac{9}{24}$$
$$y = \frac{16}{24}$$
$$y = \frac{2}{3}$$

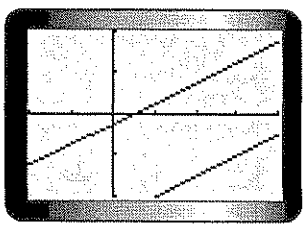
This system has exactly one solution, $\left(\frac{5}{6}, \frac{2}{3}\right)$. Graphing the system on a graphing calculator confirms this result.



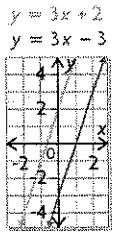
h) No solution. The equations are:
(1) $2x - 5 = 4y$
(2) $0.01x - 0.02y = 0.25$
Multiply equation (2) by 200.
(3) $2x - 4y = 50$
Subtract equation (1) from equation (3).

$$-4y + 5 = 50 - 4y$$
$$5 = 50$$

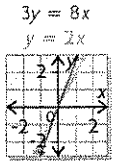
This system has a solution whenever $5 = 50$. This is never true, however, and so this system has no solutions. Graphing the system on a graphing calculator confirms this result.



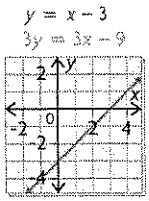
4. For example,
a) Choose two equations with the same coefficients, but differing constants.
(1) $y = 3x + 2$
(2) $y = 3x - 3$
Subtract equation (2) from equation (1).
$0 = 5$
This system has a solution whenever $5 = 50$. This is never true, however, and so this system has no solutions. Graphing the system confirms this result.



b) Choose two equations with coefficients that are not equal to multiples of one another.
(1) $y = 2x$
(2) $3y = 8x$
Substitute equation (1) into equation (2).
$3(2x) = 8x$
$6x = 8x$
$2x = 0$
$x = 0$
Substitute this value into equation (1) to solve for y.
$y = 2x$
$y = 2(0)$
$y = 0$
This system has exactly one solution, (0, 0). Graphing the system confirms this result.



c) Choose two equations that are multiples of one another.
(1) $y = x - 3$
(2) $3y = 3x - 9$
Substitute equation (1) into equation (2).
$3(x - 3) = 3x - 9$
$3x - 9 = 3x - 9$
$0 = 0$
This system has a solution whenever $0 = 0$. This is always true, however, and so this system has infinitely many solutions. Graphing the system confirms this result.



5. For example, if the coefficients and constants in both equations are multiplied by the same amount then there is an infinite number of solutions. If the coefficients and constants in both equations are not multiplied by the same amount then there is one solution. If the coefficients in both equations are multiplied by the same amount, but the constants are not, then there is no solution.
6. The equations are:
(1) $3x - 5y = 20$
(2) $18x = 30y + 72$
Multiply equation (1) by 6.
(3) $18x - 30y = 120$
Subtract equation (2) from equation (3).
$-30y = -30y + 48$
$0 = 48$
This system has a solution whenever $0 = 48$. This is never true, however, so this system has no solutions. The controller need not alter the paths of the aircraft, since the system has no solutions: the planes will never collide.

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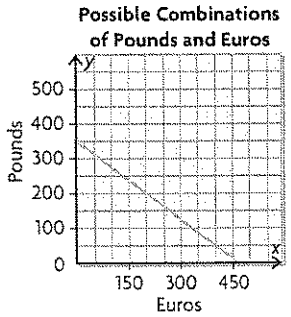
1. For example, Sheila could use a table to show some of the possible combinations. In each case, she must make sure that the total cost adds to $700.00.

Euros		Pounds		Total Cost ($)
Amount	Cost ($)	Amount	Cost ($)	
0.00	0.00	350.00	700.00	700.00
200.00	300.00	200.00	400.00	700.00
400.00	600.00	50.00	100.00	700.00
466.67	700.00	0.00	0.00	700.00

Sheila could also represent the possible combinations by using an equation. Let x represent the amount of money she desires to be in euros, and let y represent the amount of money she desires to be in pounds. Since each euro costs $1.50, $1.5x$ represents the amount of money, in dollars, that she needs to receive x euros. Since each pound costs $2.00, $2y$ represents the amount of money, in dollars, that she needs to receive y pounds. Sheila has $700.00, so the linear relationship relating these amounts is: $1.5x + 2y = 700$
Finally, Sheila could represent the combinations on a graph. She could do so either by using her table of values to plot points, or by using her equation to determine the slope, x-intercepts, and/or y-intercepts. The graph should look something like this:



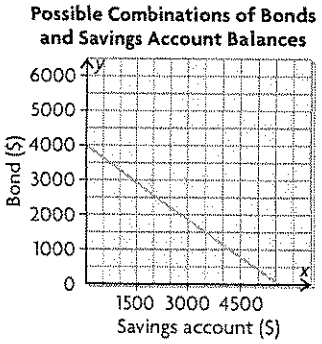
2. Let x represent the amount of money invested in the savings account, and let y represent the amount of money invested in the government bond. The money in the savings account accumulated 2.5% of its value in interest, or $0.025x$. The money in the government bond accumulated 3.5% of its value, or $0.035y$. The total interest earned was \$140. This relationship can be expressed as:

$0.025x + 0.035y = 140$

Rearrange this equation in order to graph.

$0.035y = -0.025x + 140$

$$y = -\frac{5}{7}x + 4000$$



3. a) The coordinates (5, 25.5) reveal that, after 5 hours of driving, 25.5 L of fuel remained in the tank. Since the trip started at 8:15 a.m., the time was 1:15 p.m. at this point.

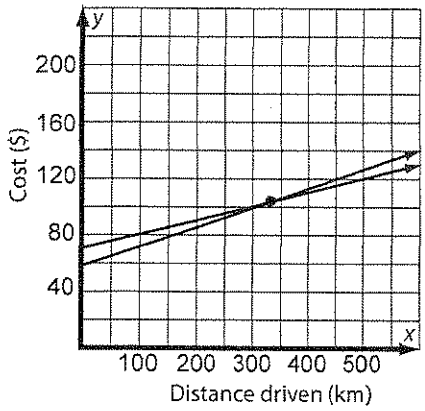
b) At 11:45 a.m., Gary had been driving for 3.5 hours. For example, upon examining the graph, it appears that the tank had approximately 37 L of fuel left at this point.

c) For example, upon inspection of the graph, it appears that Gary had been driving for approximately 7.5 hours when level of fuel in the tank dropped to 6 L. Since his trip started at 8:15 a.m., this means that the time was about 3:45 p.m. at this point.

4. For example, make a table that indicates the cost of renting a car for a day as a function of the distance driven.

Distance Driven (km)	Readycars Price ($)	Bestcars Price ($)
0	59.00	69.00
50	66.00	74.50
100	73.00	80.00
150	80.00	85.50
200	87.00	91.00
250	94.00	96.50
300	101.00	102.00
350	108.00	107.50
400	115.00	113.00

Alternatively, draw a graph to compare the costs visually.



Finally, these relationships can be expressed using equations. Let C represent the cost, and x the distance driven.

Readycars: $C = 0.14x + 59$

Bestcars: $C = 0.11x + 69$

Upon inspection of the graph and the table, it appears that Readycars offers a better price when driving less than about 333 km each day. For driving distances greater than that, Bestcars is the best choice.

5. a) The equations are:

(1) $x + y = 2$

(2) $x = 2y + 2$

Determine slopes and intercepts in order to graph.

Equation (1):

(1) $x + y = 2$

Rearrange Equation (1).

$y = -x + 2$

This relation has a slope of -1 and a y-intercept of 2.

Equation (2):

(2) $x = 2y + 2$

Solve for the y-intercept. Let $x = 0$.

$0 = 2y + 2$

$2y = -2$

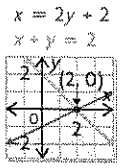
$y = -1$

Solve for the x-intercept. Let $y = 0$.

$x = 2(0) + 2$

$x = 2$

Graph the relations.



The point of intersection is at (2, 0); so, this point is the solution to this linear system.

b) The equations are:

(1) $y - x = 1$

(2) $2x - y = 1$

Determine slopes and intercepts in order to graph.

Equation (1):

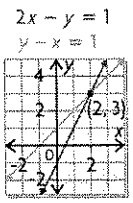
(1) $y - x = 1$

Rearrange equation (1).

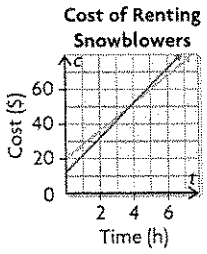
$y = x + 1$

This relation has a slope of 1, and a y-intercept of 1.

Equation (2):
(2) $2x - y = 1$
Solve for y to obtain the y-intercept. Let $x = 0$.
$-y = 1$
$y = -1$
Solve for x to obtain the x-intercept. Let $y = 0$.
$2x = 1$
$x = \frac{1}{2}$
The graph looks like the following:



The point of intersection is at (2, 3); so, this point is the solution to this linear system.
6. Let C represent the total cost of the rental and t represent the number of hours for which the snowblower is rented.
a) Tools-R-Us:
(1) $C = 20 + 8t$
b) Randy's Rentals:
(2) $C = 12 + 10t$
c) Equation (1) has a slope of 8, and $C = 20$ when $t = 0$. Equation (2) has a slope of 10, and $C = 12$ when $t = 0$. Graph the relations.



The lines appear to intersect at (4, 52), which is the solution to the system.
d) In this situation, the point of intersection represents the point where both companies charge the same amount. Both companies charge $52 for a 4 h rental.
7. a) The equations are:
(1) $2x + 3y = 7$
(2) $-2x - 1 = y$
Substitute equation (2) into equation (1) and solve for x.
$2x + 3(-2x - 1) = 7$
$2x - 6x - 3 = 7$
$-4x = 10$
$x = -\frac{5}{2}$
Substitute this value into equation (2) to solve for y.
$-2x - 1 = y$
$-2\left(-\frac{5}{2}\right) - 1 = y$
$5 - 1 = y$
$y = 4$
The solution is $\left(-\frac{5}{2}, 4\right)$.
b) The equations are:
(1) $3x - 4y = 5$
(2) $x - y = 5$
Rearrange equation (2).
(3) $y = x - 5$
Substitute equation (3) into equation (1) and solve for x.
$3x - 4(x - 5) = 5$
$3x - 4x + 20 = 5$
$-x + 20 = 5$
$-x = -15$
$x = 15$
Substitute this value into equation (3) to solve for y.
$y = x - 5$
$y = 15 - 5$
$y = 10$
The solution is (15, 10).
c) The equations are:
(1) $5x + 2y = 18$
(2) $2x + 3y = 16$
Rearrange equation (1) and divide it by 2.
(3) $y = -\frac{5}{2}x + 9$
Substitute equation (3) into equation (2) and solve for x.
$2x + 3\left(-\frac{5}{2}x + 9\right) = 16$
$2x - \frac{15}{2}x + 27 = 16$
$-\frac{11}{2}x = -11$
$x = 2$
Substitute this value into equation (3) to solve for y.
$y = -\frac{5}{2}x + 9$
$y = -\frac{5}{2}(2) + 9$
$y = -5 + 9$
$y = 4$
The solution is (2, 4).
d) The equations are:
(1) $9 = 6x - 3y$ (2) $4x - 3y = 5$
Rearrange equation (2) to isolate $3y$.
(3) $3y = 4x - 5$
Substitute equation (3) into equation (1) and solve for x.
$9 = 6x - (4x - 5)$
$9 = 6x - 4x + 5$
$9 = 2x + 5$
$2x = 4$
$x = 2$
Substitute this value into equation (3) to solve for y.
$3y = 4x - 5$
$3y = 4(2) - 5$
$3y = 8 - 5$
$3y = 3$
$y = 1$
The solution is (2, 1).

8. First, develop an equation to represent the total cost of Courtney's membership with respect to how many months she has been a member. Let C represent the total cost, and let t represent the number of months. Let m represent the monthly fee, and let r represent the registration fee. The equation is:
$C = mt + r$
Substitute the known costs into this equation to develop a linear system of two equations.
(1) $315 = 3m + r$ (2) $535 = 7m + r$
Rearrange equation (2) to isolate r.
(3) $r = -7m + 535$
Substitute equation (3) into equation (1) and solve for m.
$315 = 3m + (-7m + 535)$
$315 = -4m + 535$
$4m = 220$
$m = 55$
Substitute this value into equation (3) to solve for r.
$r = -7m + 535$
$r = -7(55) + 535$
$r = -385 + 535$
$r = 150$
Courtney paid a \$150 registration fee and pays monthly fees of \$55.

9. a) The formula for the perimeter of a rectangle is given by $P = 2(l + w)$, where P represents the perimeter, l the length, and w the width. In this case:
$2(l + w) = 40$
(1) $2l + 2w = 40$
Also, if the length is 2 m greater than the width, then:
(2) $l = w + 2$
b) Substitute equation (2) into (1), and solve for w.
$2(w + 2) + 2w = 40$
$2w + 4 + 2w = 40$
$4w = 36$
$w = 9$
Substitute this value into equation (2) to solve for l.
$l = w + 2$
$l = 9 + 2$
$l = 11$
The solution to this system is $l = 11$ and $w = 9$.
c) The rectangle is 11 m long and 9 m wide.

10. a) From inspection of the graph, observe that the point of intersection for the two lines is (–3, –2). This is also the solution to any equivalent linear system. Substitute this point into systems A and B to check whether it is a solution to each system.
System A:
The equations are:
(1) $2x - 5y = 4$
(2) $-x + y = 1$
Check whether the point (–3, –2) satisfies equation (1).
LS $= 2x - 5y$
$= 2(-3) - 5(-2)$
$= -6 + 10$
$= 4$
RS $= 4$
LS $=$ RS
The point (–3, –2) satisfies equation (1).
Check whether the point (–3, –2) satisfies equation (2).
LS $= -x + y$
$= -(-3) + (-2)$
$= 3 - 2$
$= 1$
RS $= 1$
LS $=$ RS
The point (–3, –2) satisfies equation (2).
Since (–3, –2) satisfies both equations in the system, it is a solution for the system. System A is equivalent to the given system.
System B:
The equations are:
(1) $x - 3y = -1$
(2) $2x + y = 4$
Check whether the point (–3, –2) satisfies equation (1).
LS $= x - 3y$
$= (-3) - 3(-2)$
$= -3 + 6$
$= 3$
RS $= -1$
LS $\neq$ RS
The point (–3, –2) does not satisfy equation (1). Since system B does not share the solution of the given system, system B is not equivalent to it.
b) For example, take the equations from system A.
(1) $2x - 5y = 4$ (2) $-x + y = 1$
Add equation (1) and equation (2).
(3) $x - 4y = 5$
Subtract equation (2) from equation (1).
(4) $3x - 6y = 3$
Equations (3) and (4) form an equivalent linear system.
c) For example, take the equations from system A.
(1) $2x - 5y = 4$ (2) $-x + y = 1$
Multiply equation (1) by 2.
(3) $4x - 10y = 8$
Multiply equation (2) by 3.
(4) $-3x + 3y = 3$
Equations (3) and (4) form an equivalent linear system.

11. a) For example, these are the equations.
(1) $-2x - 3y = 5$ (2) $3x - y = 9$
Add equations (1) and (2).
(3) $x - 4y = 14$
Subtract equation (2) from equation (1).
(4) $-5x - 2y = -4$
Equations (3) and (4) form an equivalent system. Call it system A. To form a second system:
Multiply equation (1) by 3.
(5) $-6x - 9y = 15$
Multiply equation (2) by –1.
(6) $-3x + y = -9$
Equations (5) and (6) form an equivalent system as well. Call it system B.
b) Use substitution to solve the given system. The equations are: (1) $-2x - 3y = 5$ (2) $3x - y = 9$
Rearrange equation (2).
(7) $y = 3x - 9$
Substitute equation (7) into equation (1).
$-2x - 3(3x - 9) = 5$
$-2x - 9x + 27 = 5$
$-11x = -22$
$x = 2$

Substitute this value into equation (7) to solve for y.

$$\begin{aligned} y &= 3x - 9 \\ y &= 3(2) - 9 \\ y &= 6 - 9 \\ y &= -3 \end{aligned}$$

The solution to this system is $(2, -3)$. Check whether it is the solution to the other two systems by checking whether it satisfies their equations.

System A:

Check if $(2, -3)$ satisfies equation (3).

$$\begin{aligned} \text{LS} &= x - 4y \\ &= (2) - 4(-3) \\ &= 2 + 12 \\ &= 14 \\ \text{RS} &= 14 \\ \text{LS} &= \text{RS} \end{aligned}$$

The point $(2, -3)$ satisfies equation (3).

$$\begin{aligned} \text{LS} &= -5x - 2y \\ &= -5(2) - 2(-3) \\ &= -10 + 6 \\ &= -4 \\ \text{RS} &= -4 \\ \text{LS} &= \text{RS} \end{aligned}$$

The point $(2, -3)$ satisfies equation (4) as well. It is the solution to system A as well as the given system; so, these are equivalent systems.

System B:

Check if $(2, -3)$ satisfies equation (5).

$$\begin{aligned} \text{LS} &= -6x - 9y \\ &= -6(2) - 9(-3) \\ &= -12 + 27 \\ &= 15 \\ \text{RS} &= 15 \\ \text{LS} &= \text{RS} \end{aligned}$$

The point $(2, -3)$ satisfies equation (5).

$$\begin{aligned} \text{LS} &= -3x + y \\ &= -3(2) + (-3) \\ &= -6 - 3 \\ &= -9 \\ \text{RS} &= -9 \\ \text{LS} &= \text{RS} \end{aligned}$$

The point $(2, -3)$ satisfies equation (6) as well. It is also the solution to system B; so, these are all equivalent systems.

12. a) The equations are:

(1) $2x - 3y = 13$

(2) $5x - y = 13$

Multiply equation (2) by 3.

(3) $15x - 3y = 39$

Subtract equation (1) from equation (3) and solve for x.

$$\begin{aligned} 13x &= 26 \\ x &= 2 \end{aligned}$$

Substitute this value into equation (1) to solve for y.

$$\begin{aligned} 2x - 3y &= 13 \\ 2(2) - 3y &= 13 \\ 4 - 3y &= 13 \\ -3y &= 9 \\ y &= -3 \end{aligned}$$

The point $(2, -3)$ is the solution to this system.

b) The equations are:

(1) $x - 3y = 0$

(2) $3x - 2y = -7$

Multiply equation (1) by 3.

(3) $3x - 9y = 0$

Subtract equation (2) from equation (3) and solve for y.

$$\begin{aligned} -7y &= 7 \\ y &= -1 \end{aligned}$$

Substitute this value into equation (1) to solve for x.

$$\begin{aligned} x - 3y &= 0 \\ x - 3(-1) &= 0 \\ x + 3 &= 0 \\ x &= -3 \end{aligned}$$

The point $(-3, -1)$ is the solution to this system.

c) The equations are:

(1) $3x + 21 = 5y$

(2) $4y + 6 = -9x$

Rearrange equations (1) and (2).

(3) $3x - 5y = -21$

(4) $9x + 4y = -6$

Multiply equation (3) by 3.

(5) $9x - 15y = -63$

Subtract equation (5) from equation (4) and solve for y.

$$\begin{aligned} 19y &= 57 \\ y &= 3 \end{aligned}$$

Substitute this value into equation (1) to solve for x.

$$\begin{aligned} 3x + 21 &= 5y \\ 3x + 21 &= 5(3) \\ 3x + 21 &= 15 \\ 3x &= -6 \\ x &= -2 \end{aligned}$$

The point $(-2, 3)$ is the solution to this system.

d) The equations are:

(1) $x - \frac{1}{3}y = -1$

(2) $\frac{2}{3}x - \frac{1}{4}y = -1$

Multiply equation (1) by 9, and multiply equation (2) by 12.

(3) $9x - 3y = -9$

(4) $8x - 3y = -12$

Subtract equation (4) from equation (3) and solve for x.

$$x = 3$$

Substitute this value into equation (3) to solve for y.

$$\begin{aligned} 9x - 3y &= -9 \\ 9(3) - 3y &= -9 \\ 27 - 3y &= -9 \\ -3y &= -36 \\ y &= 12 \end{aligned}$$

The point $(3, 12)$ is the solution to this system.

13. If Lyle needs 200 g of chocolate for his recipe, and 86% of that is cocoa, then Lyle needs 0.86×200 g $= 172$ g of cocoa. Let x represent the number of grams of 99% cocoa that he uses, and let y represent the number of grams of 70% cocoa that he uses. The $0.99x$ represents the mass of cocoa from x grams of the chocolate that is 99% cocoa, and $0.70y$ represents the mass of cocoa from y grams of the chocolate that is 70% cocoa. Then:

(1) $0.99x + 0.70y = 172$

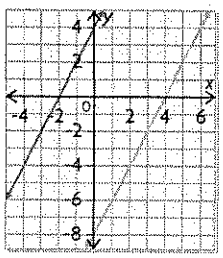
Also, the total mass of chocolate must add to 200 g. So:
(2) $x + y = 200$
Multiply equation (2) by 0.99.
(3) $0.99x + 0.99y = 198$
Subtract equation (1) from equation (3) and solve for y.
$0.29y = 26$
$y \doteq 90$
Substitute this value into equation (2) to solve for x.
$x + y = 200$
$x + 90 \doteq 200$
$x \doteq 110$
Lyle needs about 110 g of the chocolate that is 99% cocoa and 90 g of the chocolate that is 70% cocoa.

14. Let x represent the cost of a desk, and let y represent the cost of a chalkboard. The following linear relations express what is known about the prices of desks and chalkboards.
(1) $35x + 3y = 2082$
(2) $40x + 2y = 2238$
Multiply equation (1) by 2, and equation (2) by 3.
(3) $70x + 6y = 4164$
(4) $120x + 6y = 6714$
Subtract equation (1) from equation (4) and solve for x.
$50x = 2550$
$x = 51$
Substitute this value into equation (2) to solve for y.
$40x + 2y = 2238$
$40(51) + 2y = 2238$
$2040 + 2y = 2238$
$2y = 198$
$y = 99$
A desk costs \$51, and a chalkboard costs \$99.

15. The equations are:
(1) $2(2x - 1) - (y - 4) = 11$
(2) $3(1 - x) - 2(y - 3) = -7$
Expand and simply both equations.
Equation (1):
$4x - 2 - y + 4 = 11$
(3) $4x - y = 9$
Equation (2):
$3 - 3x - 2y + 6 = -7$
(4) $-3x - 2y = -16$
Multiply equation (3) by 2.
(5) $8x - 2y = 18$
Subtract equation (4) from equation (5) and solve for x.
$11x = 34$
$x = \frac{34}{11}$
Substitute this value into equation (3) to solve for y.
$4x - y = 9$
$4\left(\frac{34}{11}\right) - y = 9$
$\frac{136}{11} - y = \frac{99}{11}$
$y = \frac{136}{11} - \frac{99}{11}$
$y = \frac{37}{11}$
This linear system has the solution $\left(\frac{34}{11}, \frac{37}{11}\right)$.

16. Let x represent the number of \$5 bills Juan has, and let y represent the number of \$10 bills. Juan has a total of 76 bills, and so:
(1) $x + y = 76$
Also, Juan has a total of \$580. Therefore:
(2) $5x + 10y = 580$
Divide equation (2) by 5.
(3) $x + 2y = 116$
Subtract equation (1) from equation (3) and solve for y.
$y = 40$
Substitute this value into equation (1) to solve for x.
$x + y = 76$
$x + (40) = 76$
$x = 36$
Juan has 36 five-dollar bills, and 40 ten-dollar bills.

17. a) For example,



This linear system has no solution because the lines are parallel, and so they never intersect.
b) For example, note that one line on the sketch has a slope of 2 and a y-intercept of –8., An equation representing this line is:
$y = 2x - 8$
An equivalent equation can be formed by multiplying the above equation by 3.
$3y = 6x - 24$
The other line has a slope of 2 and y-intercept of 4. An equation representing this line is:
$y = 2x + 4$
An equivalent equation can be generated by multiplying the above equation by 4.
$4y = 8x + 16$
c) These lines both have the same slope. This makes sense since the lines are parallel.

18. The two equations are:
(1) $6x + 5y = 10$
(2) $ax + 2y = b$
Multiply equation (2) by $\frac{5}{2}$ for the y terms to have the same coefficient.
(3) $\frac{5}{2}ax + 5y = \frac{5}{2}b$
For a linear system to have an infinite number of solutions, its equations must be multiples of one another. Equation (3) is the multiple of Equation (2) that produces the same coefficient for y as in Equation (1). The coefficients for x must also be the same, as well as the constant terms, if these equations are to be multiples.

Then:

$\frac{5}{2}a = 6$

$a = \frac{12}{5}$

Also:

$\frac{5}{2}b = 10$

$b = \frac{20}{5}$

$b = 4$

Then, $a = \frac{12}{5}$ or 2.4, and $b = 4$.

Chapter Self-Test, page 64

1. Let x represent the number of 500 g (or 0.5 kg) cartons, and y represent the number of 750 g (or 0.75 kg) bags. There must be a total mass of 887.5 kg. This relationship can be expressed by writing the following equation: $0.5x + 0.75y = 887.5$

Solve for the x-intercept and y-intercept to graph.

y-intercept: Let $x = 0$.

$0.5(0) + 0.75y = 887.5$

$0.75y = 887.5$

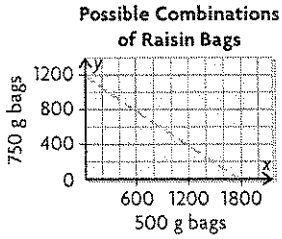
$y = 1183.33$

x-intercept: Let $y = 0$.

$0.5x + 0.75(0) = 887.5$

$0.5x = 887.5$

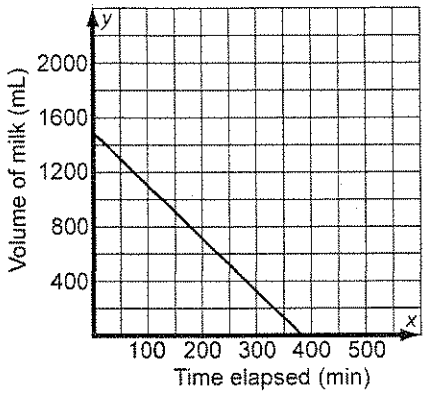
$x = 1775$



The graph above shows all the possible combinations of bags and cartons.

2. a) Let V represent the volume of milk remaining, and let t represent the amount of time that has elapsed since 8:30 a.m. The rate of change of the volume is –4 mL/min, so the slope is –4. The carton has 1500 mL of milk at 8:30 a.m., and since this is time-zero, the value of the y-intercept is 1500. The equation and graph is:

$V = 1500 - 4t$



b) For example, determine when 1 L (1000 mL) of milk remains on the graph, and draw a line down to the horizontal axis. It appears that about 120 min have elapsed at this point, or 2 h. At about 10:30 a.m., 1 L of milk remains.

c) Substitute a value of $V = 1000$ mL into the equation, and solve for t.

$V = -4t + 1500$

$1000 = -4t + 1500$

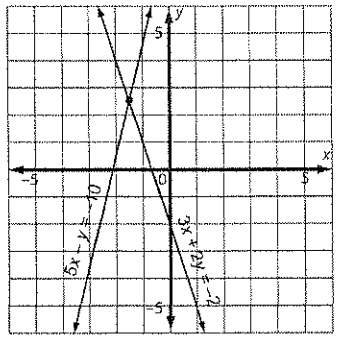
$-500 = -4t$

$t = 125$

There will be precisely 1 L of milk left after exactly 125 min, or 2 h 5 min have elapsed. The corresponding time for this is 10:35 a.m.

3. a) Graphical solution:

Graph each equation on the same grid.



From the graph, the two lines intersect at about (–1.5, 2.5).

Algebraic solution by substitution:

The two equations are:

(1) $3x + y = -2$

(2) $5x - y = -10$

Rearrange equation (1) to isolate y.

(3) $y = -3x - 2$

Substitute equation (3) into equation (2) and solve for x.

$5x - (-3x - 2) = -10$

$5x + 3x + 2 = -10$

$8x = -12$

$x = -1.5$

Solve for y.

$y = -3x - 2$

$y = -3(-1.5) - 2$

$y = 4.5 - 2$

$y = 2.5$

The solution is (–1.5, 2.5).

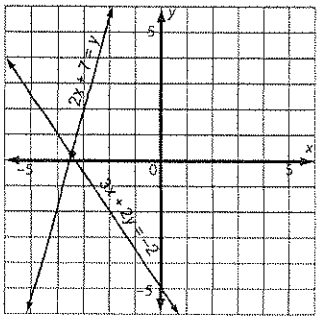
Algebraic solution by elimination:
The two equations are:
(1) $3x + y = -2$ (2) $5x - y = -10$
Add the two equations and solve for x.

$$\begin{aligned} 8x &= -12 \\ x &= -1.5 \end{aligned}$$

Solve for y.

$$\begin{aligned} y &= -3x - 2 \\ y &= -3(-1.5) - 2 \\ y &= 4.5 - 2 \\ y &= 2.5 \end{aligned}$$

The solution is (–1.5, 2.5).
b) Graphical solution:
Graph each equation on the same grid.



The two lines intersect at about $\left(\frac{-24}{7}, \frac{1}{7}\right)$.
Algebraic solution by substitution:
The two equations are:
(1) $2x + 7 = y$ (2) $-3x - 2y = 10$
Substitute equation (1) into equation (2) and solve for x.

$$\begin{aligned} -3x - 2(2x + 7) &= 10 \\ -3x - 4x - 14 &= 10 \\ -7x &= 24 \\ x &= -\frac{24}{7} \end{aligned}$$

Solve for y.

$$y = 2x + 7$$

$$y = 2\left(-\frac{24}{7}\right) + 7$$

$$y = -\frac{48}{7} + \frac{49}{7}$$

$$y = \frac{1}{7}$$

The solution is $\left(-\frac{24}{7}, \frac{1}{7}\right)$.
Algebraic solution by elimination:
The two equations are:
(1) $2x + 7 = y$
(2) $-3x - 2y = 10$
Rearrange equation (1) and multiply it by 2.
(3) $4x - 2y = -14$
Subtract equation (2) from equation (3) and solve for x.

$$7x = -24$$

$$x = -\frac{24}{7}$$

Solve for y.

$$y = 2x + 7$$

$$y = 2\left(-\frac{24}{7}\right) + 7$$

$$y = -\frac{48}{7} + \frac{49}{7}$$

$$y = \frac{1}{7}$$

The solution is $\left(-\frac{24}{7}, \frac{1}{7}\right)$.
c) Graphical solution:
Graph each equation on the same grid.
From the graph, the two lines intersect at about (2, –3).
Algebraic solution by substitution:
The two equations are:
(1) $3x + 5y = -9$
(2) $2x - y = 7$
Rearrange equation (2) to isolate y.
(3) $y = 2x - 7$
Substitute equation (3) into equation (1) and solve for x.

$$\begin{aligned} 3x + 5(2x - 7) &= -9 \\ 3x + 10x - 35 &= -9 \\ 13x &= 26 \\ x &= 2 \end{aligned}$$

Solve for y.

$$\begin{aligned} y &= 2x - 7 \\ &= 2(2) - 7 \\ &= 4 - 7 \\ &= -3 \end{aligned}$$

The solution is (2, –3).
Algebraic solution by elimination:
The two equations are:
(1) $3x + 5y = -9$
(2) $2x - y = 7$
Multiply equation (2) by 5.
(3) $10x - 5y = 35$
Add equation (3) and equation (1) and solve for x.

$$\begin{aligned} 13x &= 26 \\ x &= 2 \end{aligned}$$

Solve for y.

$$\begin{aligned} 2x - y &= 7 \\ 2(2) - y &= 7 \\ y &= 4 - 7 \\ y &= -3 \end{aligned}$$

The solution is (2, –3).

4. Let x represent the mass of 85% gold Shannon uses and y represent the mass of 70% gold she uses. The sum of these two masses must add to 20 g. Express this relationship using the following linear equation:
(1) $x + y = 20$
Also, she requires 80% of the pendant to be pure gold. This means that of the pendant's 20 g mass, 0.8×20 g = 16 g are pure gold. Then, the masses of pure gold taken from the broken jewellery must also add to this total. Express this relationship using the following linear equation:
(2) $0.85x + 0.70y = 16$
Solve the system of linear equations to determine the quantity of each alloy that Shannon should use.
Multiply equation (1) by 0.70.
(3) $0.70x + 0.70y = 14$
Subtract equation (3) from equation (2).

$$\begin{aligned} 0.15x &= 2 \\ x &\doteq 13.33 \end{aligned}$$

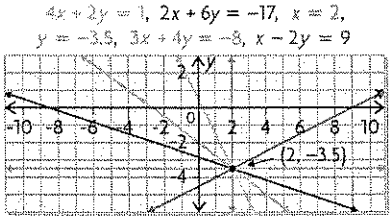
Solve for y.

$$\begin{aligned} x + y &= 20 \\ y &\doteq 20 - 13.33 \\ y &\doteq 6.67 \end{aligned}$$

Shannon should use about 13.33 g of 85% gold, and about 6.67 g of 70% gold to make her pendant.

5. For example, at the point (x, y) which represents a solution of a linear system, both sides of an equation in that system must be equal. Adding or subtracting these equations is the same as adding or subtracting constants to both sides of an equation: the solution will remain the same.

6. a) The two equations are:
(1) $3x + 4y = -8$
(2) $x - 2y = 9$
For example, add and subtract the two equations to produce one equivalent linear system.
(1) + (2): $4x + 2y = 1$
(1) – (2): $2x + 6y = -17$
We could also add multiply an equation by a constant and then add the two equations to yield another equivalent linear system.
(1) + 2 × (2): $5x = 10$, or equivalently, $x = 2$.
(1) – 3 × (2): $10y = -35$, or equivalently, $y = -3.5$
Thus, we have another equivalent linear system.
b) Sketch each system on a graph. Verify that they all intersect at (2, –3.5).



Since the systems intersect at the same point, they are equivalent.

7. Let x represent the amount of time Kim spent biking on pavement and y represent the amount of time Kim spent biking off-road. He spent a total of 2 h biking that day, which can be represented by this linear equation:
(1) $x + y = 2$
Also, he cycled a total distance of 41 km. Distance is equal to speed multiplied by time, since speed represents the amount of distance covered in a unit of time. The distance Kim biked on pavement is equal to $25x$, and the distance he biked off-road is equal to $10y$. This distance relationship can be expressed as the following linear equation:
(2) $25x + 10y = 41$
Use elimination to solve this system of linear equations for y, the distance Kim biked off-road. Multiply equation (1) by 25.
(3) $25x + 25y = 50$
Subtract equation (2) from equation (3).

$$\begin{aligned} 15y &= 9 \\ y &= 0.6 \end{aligned}$$

Recall that y represents the time Kim spent biking off-road, not the distance. $10y$ represents the distance Kim biked off-road.

$$\begin{aligned} 10y &= 10(0.6) \\ &= 6 \end{aligned}$$

Kim cycled 6 km off-road that day.

8. Let x represent the amount of money that Betty invested in her savings account and y the amount of money that she invested in a government bond. She invested $4200 dollars total, so:
(1) $x + y = 4200$
Also, she knows the sum of the interest she earned on each investment is 174 dollars. One part of that is3.5% of the amount in her savings account, and the other part is 4.5% of the money that she invested in the government bond. This can be expressed using the following linear relationship:
(2) $0.035x + 0.045y = 174$
The above system of linear equations can be solved using elimination. First, multiply equation (1) by 0.045.
(3) $0.045x + 0.045y = 189$
Subtract equation (2) from equation (3) and solve for x.

$$\begin{aligned} 0.010x &= 15 \\ x &= 1500 \end{aligned}$$

Substitute into equation (1) to solve for y.

$$\begin{aligned} x + y &= 4200 \\ (1500) + y &= 4200 \\ y &= 4200 - 1500 \\ y &= 2700 \end{aligned}$$

Betty deposited $1500 into her savings account and invested $2700 in a government bond.

9. For example, try to solve this system using elimination.
(1) $15 - 6y = 9x$
(2) $3x + 2y = 8$
Multiply equation (2) by 3.
(3) $9x + 6y = 24$
Add equation (1) and equation (3).
$9x + 15 = 9x + 24$
$15 = 24$
This system has a solution whenever $15 = 24$. Clearly, 15 is never equal to 24, so this is never true. This system has no solution.

Chapter Task, page 65

A. Pear and pineapple: Let u represent the grams of pear and v represent the grams of pineapple in Stuart's combination.
(1) $0.04u + 0.25v = 25$ (2) $u + v = 250$
Banana and blueberry: Let w represent the grams of banana and x represent the grams of blueberry in Stuart's combination.
(3) $0.10w + 0.10x = 25$ (4) $w + x = 250$
Apple and cranberry: Let y represent the grams of pear and z represent the grams of pineapple in Stuart's combination.
(5) $0.15y + 0.15z = 25$ (6) $y + z = 250$
B. Pear and pineapple:
(1) $0.04u + 0.25v = 25$
(2) $u + v = 250$
Multiply equation (1) by 4.
(7) $0.16u + v = 100$
Subtract equation (7) from equation (2).
$0.84u = 150$
$u \doteq 178.6$
Solve for v.
$u + v = 250$
$v \doteq 250 - 178.6$
$v \doteq 71.4$
Banana and blueberry:
(3) $0.10w + 0.10x = 25$ (4) $w + x = 250$
Multiply equation (3) by 10.
(8) $w + x = 250$
Equations (4) and (8) are equivalent, so any combination of banana and blueberry that adds up to 250 g is suitable.
Apple and cranberry:
(5) $0.15y + 0.15z = 25$ (6) $y + z = 250$
Multiply equation (6) by 0.15.
(9) $0.15y + 0.15z = 37.5$
Equations (5) and (9) are similar, but have different constants, so there is no solution to this system.

C. Pear and pineapple: This system has only one solution as shown in part B: 178.6 g of pear, and 71.4 g of pineapple.
Banana and blueberry: The equations in part B are equivalent; so, this system has infinite solutions. Any combination of banana and blueberry with a total mass of 250 g will contain 25 mg of vitamin C.
Apple and cranberry: The equations in part B are similar, but have different constants. This system has no solutions. No 250 g combination of apple and cranberry will provide 25 mg of vitamin C.
D. For example, grapefruit and strawberry: Let a represent the grams of grapefruit, and b represent the grams of strawberry in the combination.
(10) $0.40a + 0.60b = 25$ (11) $a + b = 250$
Multiply equation (11) by 0.40.
(12) $0.40a + 0.40b = 100$
Subtract equation (10) from equation (12).
$-0.20b = 75$
$b = -375$
Solve for a.
$a + b = 250$
$a = 250 - (-275)$
$a = 525$
This system of linear equations has a solution; however, it is not a physical solution. It would require –375 g of strawberry and 525 g of grapefruit. Negative mass does not exist; so, this is impossible.
Cherry and orange: Let c represent the grams of cherry and d represent the grams of orange in the combination.
(13) $0.10c + 0.49d = 25$ (14) $c + d = 250$
Multiply equation (13) by 10.
(15) $c + 4.9d = 250$
Subtract equation (14) from equation (15).
$3.9d = 0$
$d = 0$
Solve for c.
$c + d = 250$
$c = 250 - 0$
$c = 250$
This system of linear equations has a solution; however, the solution is to use precisely 250 g of cherry and no orange. This isn't a combination of fruit.
Pear and kiwi: Let e represent the grams of pear, and f represent the grams of kiwi in the combination.
(16) $0.04e + 0.70f = 25$
(17) $e + f = 250$

Multiply equation (17) by 0.04.
(18) $0.04e + 0.04f = 10$
Subtract equation (18) from equation (16).
$0.66f = 15$
$f = 22.7$
Solve for e.
$e + f = 250$
$e = 250 - 22.7$
$e = 227.3$
This system of linear equations has a solution. The solution is to mix 227.3 g of pear and 22.7 g of kiwi.
E. A total of 250 g of fruit is required to produce 25 mg of vitamin C. Each gram of fruit must average 0.10 mg of vitamin C, since $25 \div 250 = 0.10$ mg.
Any combination of fruit that cannot achieve this average is an impossibility. That means that if Stuart takes two fruits that have more than 0.10 mg of vitamin C per gram, then he will surely exceed the required amount. Examples illustrating this are the combination of apple (0.15 mg) and cranberry (0.15 mg), which has no solution, and the combination grapefruit (0.40 mg) and strawberry (0.60 mg), which has a solution that is impossible in real life.
Three fruits exist that have exactly 0.10 mg of vitamin C per gram: banana, blueberry, and cherry. 250 g of any of these fruit will therefore produce exactly 25 mg of vitamin C. This explains why in the cherry (0.10 mg) and orange (0.49 mg) mix, exactly 250 g of cherry and no orange were required. Combining one of banana, blueberry, or cherry with any other fruit will always produce the result where 250 g of these fruit, but none of the other, are required. However, these are not combinations of fruit. This also explains why infinite solutions for the banana and blueberry combination exist. Since 250 g of banana, blueberry, or cherry always amounts to 25 mg of vitamin C, then any combination of these fruits will, necessarily, also amount to 25 mg of vitamin C.
Finally, true combinations exist for combinations of pear and any fruit other than banana, blueberry, or cherry (which will require 250 g of only one of these fruits as before). This is true because pear (0.04 mg) is the only fruit with less than 0.10 mg of vitamin C, meaning it can be paired with any fruit that has more than 0.10 mg of vitamin C. This explains why both pear and pineapple and pear and kiwi have solutions.